On the Complexity of Probabilistic Abstract Argumentation

Bettina Fazzinga, Sergio Flesca, Francesco Parisi
DIMES - University of Calabria, ITALY

Abstract Argumentation

Abstract Argumentation Framework (AAF) is simple but powerful argumentation framework proposed in [1].

– It allows representing dialogues, making decisions, and handling inconsistency and uncertainty;
– An AAF is a pair $\langle A, D\rangle$ consisting of a set A of *arguments*, and of a binary relation D over A, called *defeat* (or, equivalently, *attack*) relation;
– An argument is an abstract entity that may attack and/or be attacked by other arguments;
– An AAF can be viewed as a direct graph, whose nodes are arguments and whose edges are attacks.

Semantics for AAF

Several semantics for AAFs have been proposed to identify "reasonable" sets of arguments, called *extensions*.

- A semantics corresponds to some properties which "certify" whether a set of arguments can be profitably used to support a point of view in a discussion;
- A set S of arguments is *conflict-free* if there are no $a, b \in S$ such that a *defeats* b;
- An argument a is *acceptable* w.r.t. S iff $\forall b \in A$ such that b *defeats* a, there is $c \in S$ such that c *defeats* b.

A set $S \subseteq A$ of arguments is said to be:

1) an *admissible* extension iff S is conflict-free and all its arguments are acceptable w.r.t. S;
2) a *stable* extension iff S is conflict-free and S defeats each argument in $A \setminus S$;
3) a *complete* extension iff S is admissible and S contains all the arguments that are acceptable w.r.t. S;
4) a *grounded* extension iff S is a minimal (w.r.t. $\subseteq$) complete set of arguments;
5) a *preferred* extension iff S is a maximal (w.r.t. $\subseteq$) admissible set of arguments;
6) an *ideal* extension [a] iff S is admissible and S is contained in every preferred set of arguments.

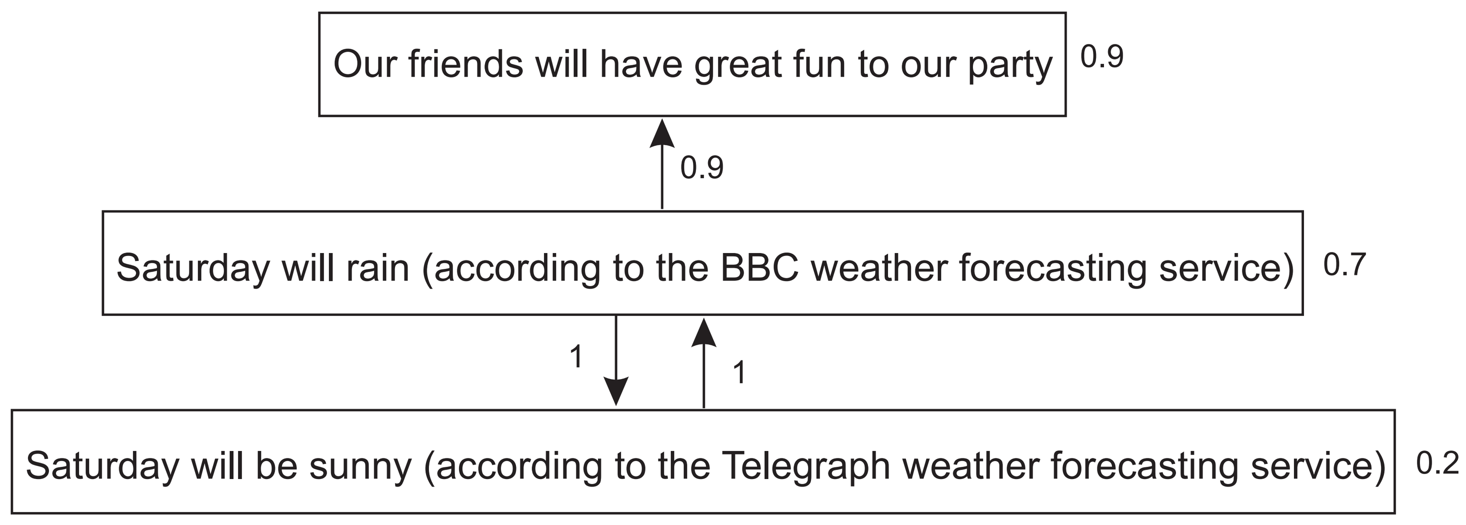
[a] In the literature, it is also referred to as *ideal set*.

Complexity of $\text{Ver}^{sem}(S)$

For the above-mentioned semantics, the problem $\text{Ver}^{sem}(S)$ of verifying whether a set S of arguments is an extension according to semantics *sem* has been addressed in [2, 3], where it is shown that the complexity of $\text{Ver}^{sem}(S)$ is as reported in the second column of the table in Section *Contributions*.

Probabilistic Argumentation Framework

In the real world, arguments and defeats are often uncertain. *Probabilistic Argumentation Framework* (PrAF) [7] associates both arguments and defeats with probabilities [a]:



– The meaning of a PrAF is based on the notion of *possible world*;
– A possible world represents a (deterministic) scenario consisting of some subset of the arguments and defeats of the PrAF (that is, a possible world can be viewed as an AAF);
– A PrAF admits a unique probability distribution over the set of possible worlds [b];
– The probability $Pr^{sem}(S)$ that a set S of arguments is an extension according to a given semantics *sem* is defined as the sum of the probabilities of the possible worlds w for which S is an extension according to *sem*.

[a] The issue of how to assign probabilities to arguments and defeats in abstract argumentation, with particular reference to the PrAF proposed in [7], has been investigated in [5, 6].
[b] Arguments are viewed as pairwise independent probabilistic events, while each defeat is viewed as a probabilistic event conditioned by the occurrence of the arguments it relates, but independent from any other event.

The problem: what is the complexity of $\text{Prob}^{sem}(S)$?

$\text{Prob}^{sem}(S)$ is the problem of *computing the probability* $Pr^{sem}(S)$ *that a set* S *of arguments is an extension according to a given semantics sem*.

Contributions

The complexity of $\text{Prob}^{sem}(S)$, which is the probabilistic counterpart of $\text{Ver}^{sem}(S)$, is reported in (third column of) the following table:

Semantics *sem*	Complexity of $\text{Ver}^{sem}(S)$	Complexity of $\text{Prob}^{sem}(S)$
admissible	*PTIME*	*PTIME*
stable	*PTIME*	*PTIME*
complete	*PTIME*	$FP^{\#P}$-complete
grounded	*PTIME*	$FP^{\#P}$-complete
preferred	*coNP*-complete	$FP^{\#P}$-complete
ideal	*coNP*-complete	$FP^{\#P}$-complete

The results on the complexity of $\text{Prob}^{sem}(S)$ are interesting from two standpoints:

1) First, comparing the complexity of $\text{Prob}^{sem}(S)$ with that of its deterministic counterpart $\text{Ver}^{sem}(S)$ shows that (*i*) for some semantics (that is, complete and grounded) $\text{Ver}^{sem}(S)$ is tractable while $\text{Prob}^{sem}(S)$ is not; (*ii*) for other semantics (that is, admissible and stable) the two problems are both tractable; and, finally, there are semantics (that is, preferred and ideal) for which these problems are both intractable;
2) Second, our complexity analysis allows us to understand for which semantics computing $Pr^{sem}(S)$ is tractable or not. In fact, the value of $Pr^{sem}(S)$ can be determined in polynomial time for the admissible and stable semantics, while the fact that computing $Pr^{sem}(S)$ is hard for the other semantics (complete, grounded, preferred, ideal) backs the use of approximate techniques for estimating $Pr^{sem}(S)$ (such as those proposed in [4, 7]).

Selected references

[1] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artif. Intell.*, 77(2):321–358, 1995.
[2] Paul E. Dunne and Michael Wooldridge. Complexity of abstract argumentation. In *Argumentation in Artificial Intelligence*, 85–104, 2009.
[3] Paul E. Dunne. The computational complexity of ideal semantics. *Artif. Intell.*, 173(18):1559–1591, 2009.
[4] Bettina Fazzinga, Sergio Flesca, and Francesco Parisi. Efficiently Estimating the Probability of Extensions in Abstract Argumentation. In *SUM*, 106–119, 2013.
[5] Anthony Hunter. Some foundations for probabilistic abstract argumentation. In *COMMA*, 117–128, 2012.
[6] Anthony Hunter. A probabilistic approach to modelling uncertain logical arguments. *Int. J. Approx. Reasoning*, 54(1):47–81, 2013.
[7] Hengfei Li, Nir Oren, and Timothy J. Norman. Probabilistic argumentation frameworks. In *TAFA*, 1–16, 2011.