Argumentation Frameworks with Strong and Weak Constraints: Semantics and Complexity

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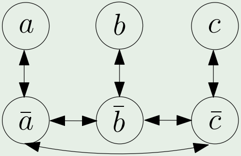
Argumentation in AI

- A general way for representing arguments and relationships (rebuttals) between them
- It allows representing dialogues, making decisions, and handling inconsistency and uncertainty

Abstract Argumentation Framework (AF) [Dung1995]: arguments are abstract entities (no attention is paid to their internal structure) that may attack and/or be attacked by other arguments

Example (A simple AF)

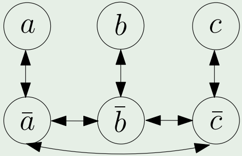


Albert (a), Betty (b) and Charlie (c) wish to attend a basketball game on Saturday evening, but only two tickets are available.

Argumentation Semantics

Several semantics have been proposed to identify "reasonable" sets of arguments (called *extensions*)

Example (AF Λ)



Semantic $\mathcal{S}$	Set of $\mathcal{S}$−extensions of Λ
complete (co)	$\{\emptyset, \{a\}, \{b\}, \{c\}, \{b,c\}, \{a,c\}, \{a,b\}, \{a,b,c\}, \{a,b,\bar{c}\}, \{a,c,\bar{b}\}, \{b,c,\bar{a}\}\}$
preferred (pr)	$\{\{a,b,c\}, \{a,b,\bar{c}\}, \{a,c,\bar{b}\}, \{b,c,\bar{a}\}\}$
semi-stable (sst)	$\{\{a,b,c\}, \{a,b,\bar{c}\}, \{a,c,\bar{b}\}, \{b,c,\bar{a}\}\}$
stable (st)	$\{\{a,b,c\}, \{a,b,\bar{c}\}, \{a,c,\bar{b}\}, \{b,c,\bar{a}\}\}$
grounded (gr)	$\{\emptyset\}$

Argument a is (resp. is not) credolously (resp. skeptically) accepted under semantics $\mathcal{S} \in \{\texttt{co}, \texttt{pr}, \texttt{st}, \texttt{sst}\}$: $CA_{\mathcal{S}}(a) = true$ (resp. $SA_{\mathcal{S}}(a) = false$).

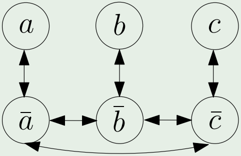
AFs with constraints (1/2)

Despite the expressive power and generality of AFs, in some cases it is difficult to accurately model domain knowledge by an AF in a natural and easy-to-understand way.

AFs with constraints (1/2)

Example

Albert, Betty and Charlie wish to attend a basketball game on Saturday evening, but only two tickets are available.



Semantic $\mathcal{S}$	Set of extensions
preferred (pr)	$\{E_1 = \{a, b, \bar{c}\}, E_2 = \{a, \bar{b}, c\}, E_3 = \{\bar{a}, b, c\}, E_4 = \{a, b, c\}\}$
stable (st)	$\{E_1 = \{a, b, \bar{c}\}, E_2 = \{a, \bar{b}, c\}, E_3 = \{\bar{a}, b, c\}, E_4 = \{a, b, c\}\}$

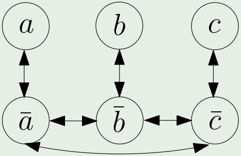
However, E_4 is not feasible, because only two tickets are available, meaning that only two people could attend the game.

To overcome such a situation, and thus providing a natural and compact way for expressing such kind of conditions, the use of constraints has been proposed.

AFs with constraints (2/2)

Example

Albert, Betty and Charlie wish to attend a basketball game on Saturday evening, but only two tickets are available.



Semantic $\mathcal{S}$	Set of extensions
preferred (pr)	$\{E_1 = \{a, b, \bar{c}\}, E_2 = \{a, \bar{b}, c\}, E_3 = \{\bar{a}, b, c\}, E_4 = \{a, b, c\}\}$
stable (st)	$\{E_1 = \{a, b, \bar{c}\}, E_2 = \{a, \bar{b}, c\}, E_3 = \{\bar{a}, b, c\}, E_4 = \{a, b, c\}\}$

The constraint $a \wedge b \wedge c \Rightarrow \texttt{false}$ can be used to state that a, b, and c are not jointly accepted, i.e., Albert, Betty and Charlie cannot attend the game together. The effect is that E_4 is discarded.

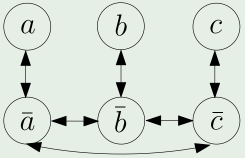
We call an AF with constraints a Constrained AF (CAF).

Introducing Weak Constraints

- Although constraints allow restricting the set of feasible solutions, they do not help in finding *best* or preferable solutions.
- If there are only two tickets available then Albert and Betty should preferably attend the game.
- This is a weak constraint which is required to be satisfied if possible.

Example (A simple WAF)

Consider a WAF obtained by adding to the previous CAF the weak constraint $\texttt{true} \rightarrow a \wedge b$, stating that is desirable that Albert and Betty attend the game together.



Semantic $\mathcal{S}$	Set of extensions
preferred ($\mathtt{pr}$)	$\{E_1 = \{a, b, \bar{c}\}, E_2 = \{a, \bar{b}, c\}, E_3 = \{\bar{a}, b, c\}, E_4 = \{a, b, c\}\}$
stable ($\mathtt{st}$)	$\{E_1 = \{a, b, \bar{c}\}, E_2 = \{a, \bar{b}, c\}, E_3 = \{\bar{a}, b, c\}, E_4 = \{a, b, c\}\}$

Then, extension E_1 is selected as the "best" preferred/stable one.

Contributions: new framework

- We propose new semantics for CAFs relying on a simple yet expressive form of constraints that are interpreted using Lukasiewicz's logic, leading to an intuitive constraints' semantics
- We introduce WAFs and propose two criteria for interpreting weak constraints: *maximal-set* ($\mathtt{ms}\mathcal{S}$) and *maximum-cardinality* ($\mathtt{mc}\mathcal{S}$)
- We investigate restricted forms of WAFs where constraints are linearly ordered (LWAF) or where constraints are expressed by denials

Contributions: complexity

- We investigate the complexity of credulous acceptance ($CA_{\mathcal{S}}$) and skeptical acceptance ($SA_{\mathcal{S}}$) for WAFs, showing that differently from strong constraints the introduction of weak constraints typically increases the complexity of one level in the polynomial hierarchy.

Framework

	AF		CAF		WAF			LWAF	NCAF	NWAF
$\mathcal{S}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$	$CA_{\mathtt{ms}\mathcal{S}}$	$SA_{\mathtt{ms}\mathcal{S}}$	$CA_{\mathtt{mc}\mathcal{S}}$ / $SA_{\mathtt{mc}\mathcal{S}}$	$CA_{\mathcal{S}}$ / $SA_{\mathcal{S}}$	$CA_{\mathcal{S}}$	$CA_{\mathtt{ms}\mathcal{S}}$
co	NP-c	P	NP-c	coNP-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c	Δ_2^P-c	NP-c	Σ_2^P-c
st	NP-c	coNP-c	NP-c	coNP-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c	Δ_2^P-c	NP-c	Σ_2^P-c
pr	NP-c	Π_2^P-c	NP-h, Σ_2^P	Π_2^P-c	Σ_2^P-h, Σ_3^P	Π_3^P-c	$\Delta_3^P[\log n]$-c	Δ_3^P-c	NP-c	Σ_2^P-c
sst	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c	Σ_3^P-c	Π_3^P-c	$\Delta_3^P[\log n]$-c	Δ_3^P-c	Σ_2^P-c	Σ_3^P-c

Outline

1. Introduction
   - Motivation
   - Contributions
2. Constrained AFs
   - Semantics and Complexity Results
3. Weak-constrained AFs
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   - Stratified WAFs
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4. Conclusions

CAF semantics (1/2)

((Strong) constraint)

Let $\mathcal{L}'_{\mathcal{A}}$ be the propositional language defined from $\mathcal{A}$ and the connectives $\wedge$, $\vee$, $\neg$, where $\mathcal{A}$ is a set of arguments.
A *(strong) constraint* is a formula of one of the following forms: (i) $\varphi \Rightarrow v$, or (ii) $v \Rightarrow \varphi$, where φ is a propositional formula in $\mathcal{L}'_{\mathcal{A}}$ and $v \in \{\mathtt{f}, \mathtt{u}, \mathtt{t}\}$.

Example

The constraint $a \wedge b \wedge c \Rightarrow \mathtt{f}$ states that at least one of the arguments a, b and c must be false, whereas $\mathtt{t} \Rightarrow a \wedge b \wedge c$ states that a, b and c must be all true.

(CAF)

A *Constrained Argumentation Framework (CAF)* is a triple $\Omega = \langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ where $\langle \mathcal{A}, \mathcal{R} \rangle$ is an AF and $\mathcal{C}$ is a set of propositional formulae built from $\mathcal{L}'_{A}$.

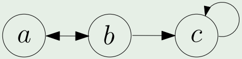
CAF semantics (2/2)

((Revised) CAF semantics)

Given a CAF $\Omega = \langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$, a set of arguments $S \subseteq \mathcal{A}$ is a complete (resp., grounded, preferred, stable, semi-stable) extension for Ω if S is a complete (resp., grounded, preferred, stable, semi-stable) extension for $\langle \mathcal{A}, \mathcal{R} \rangle$ and $S \models \mathcal{C}$.

Example (A CAF)

Consider the CAF $\Omega = \langle \{a, b, c\}, \{(a, b), (b, a), (b, c), (c, c)\}, \{\mathtt{t} \Rightarrow a \wedge b\} \rangle$.



The AF $\langle \{a, b, c\}, \{(a, b), (b, a), (b, c), (c, c)\} \rangle$ has three complete extensions, $E_1 = \emptyset$, $E_2 = \{a\}$ and $E_3 = \{b\}$, but all extensions do not satisfy the constraint stating that both a and b must belong to them. Thus Ω has no complete extensions, and thus no grounded extension.

Complexity Results

- The fact that the grounded extension may not exist for CAFs impacts on the complexity of the skeptical acceptance problem under complete semantics, which cannot be longer decided by simply looking at the grounded extension as for the case of AFs.
- Similarly, credulous acceptance under preferred semantics for CAFs can no longer be decided by checking credulous acceptance under complete semantics.

Framework

	AF		CAF	
$\mathcal{S}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$
co	NP-c	P	NP-c	coNP-c
st	NP-c	coNP-c	NP-c	coNP-c
pr	NP-c	Π_2^P-c	NP-h, Σ_2^P	Π_2^P-c
sst	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c

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4 Conclusions

WAF semantics

(Weak constrained AF)

A *Weak constrained Argumentation Framework (WAF)* is a tuple $\langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \mathcal{W} \rangle$, where $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ is a CAF and $\mathcal{W}$ is a set of weak constraints.

Example

Consider the WAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \mathcal{W} \rangle$ $a \leftrightarrow b \quad c \leftrightarrow d$
with $\mathcal{W} = \{w_1 = c \rightarrow \mathtt{f},\ w_2 = a \vee \neg a \rightarrow \mathtt{u}\}$ stating that c should preferably be false (w_1) and a should preferably be undefined (w_2).

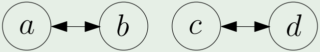
Two criteria for interpreting weak constraints

- **maximal set** criterion, considering as preferable (or *best*) extensions the ones that satisfy a maximal set of weak constraints, and
- **maximum-cardinality** criterion, considering as preferable (or *optimal*) extensions the ones that satisfy a maximal number of weak constraints.

Maximal-Set Semantics

Example (A WAF)

$\mathcal{W} = \{w_1 = c \rightarrow \texttt{f},\ w_2 = a \vee \neg a \rightarrow \texttt{u}\},\ \mathcal{C} = \emptyset$



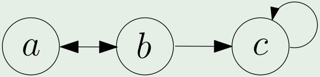
1 `co/gr` $E_0 = \{\} \models \{w_2\}$,
2 `co` $E_1 = \{a\} \models \{\}$,
3 `co` $E_2 = \{b\} \models \{\}$,
4 `co` $E_3 = \{c\} \models \{\}$,
5 `co` $E_4 = \{d\} \models \{w_1, w_2\}$,
6 `co/pr/st/sst` $E_5 = \{a, c\} \models \{\}$,
7 `co/pr/st/sst` $E_6 = \{a, d\} \models \{w_1\}$,
8 `co/pr/st/sst` $E_7 = \{b, c\} \models \{\}$ and
9 `co/pr/st/sst` $E_8 = \{b, d\} \models \{w_1\}$.

The maximal-set preferred (stable, semi-stable) extensions are E_6 and E_8, whereas there is only one maximal-set complete extension, which is E_4.

Maximum-Cardinality Semantics

Example (A WAF)

$\mathcal{W} = \{w_1 = \mathtt{t} \rightarrow a,\ w_2 = \mathtt{t} \rightarrow b, w_3 = c \rightarrow \mathtt{f}\},\ \mathcal{C} = \emptyset$



1 `co/gr` $E_1 = \{\} \models \mathcal{W}_1 = \emptyset,$

2 `co/pr` $E_2 = \{a\} \models \mathcal{W}_2 = \{w_1\},$

3 `co/pr/st/sst` $E_3 = \{b\} \models \mathcal{W}_3 = \{w_2, w_3\},$

- The only maximum-cardinality preferred extension is E_3 (as $|\mathcal{W}_3| = 2 > |\mathcal{W}_1| = 1 > |\mathcal{W}_0| = 0$).
- According to the maximal-set semantics, both E_2 and E_3 are maximal-set preferred extensions.
- Regarding the stable (and semi-stable) semantics, as there is only one extension, E_3 is both a maximal-set and a maximum-cardinality extension.

Complexity Results

	Framework						
	AF		CAF		WAF		
$\mathcal{S}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$	$CA_{\mathtt{ms}\mathcal{S}}$	$SA_{\mathtt{ms}\mathcal{S}}$	$CA_{\mathtt{mc}\mathcal{S}} / SA_{\mathtt{mc}\mathcal{S}}$
co	NP-c	P	NP-c	coNP-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c
st	NP-c	coNP-c	NP-c	coNP-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c
pr	NP-c	Π_2^P-c	NP-h, Σ_2^P	Π_2^P-c	Σ_2^P-h, Σ_3^P	Π_3^P-c	$\Delta_3^P[\log n]$-c
sst	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c	Σ_3^P-c	Π_3^P-c	$\Delta_3^P[\log n]$-c

- Differently from strong constraints the introduction of weak constraints typically increases the complexity of one level in the polynomial hierarchy.

Stratified Weak Constrained AFs

We also considered WAFs where weak constraints are partially ordered.

A *Stratified Weak constrained Argumentation Framework (SWAF)* is a WAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{C}, \mathcal{W} \rangle$ where $\mathcal{W}$ is a list of sets of weak constraints $(\mathcal{W}_1, \ldots, \mathcal{W}_n)$.

- The idea is that weak constraints are applied one stratum at a time
- Given a set S of $\mathcal{S}$-extensions of $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$, the best/optimal $\mathcal{S}$-extensions are obtained by first computing the set $S_1 \subseteq S$ which are best/optimal solutions w.r.t. $\mathcal{W}_1$, then the set $S_2 \subseteq S_1$ of $\mathcal{S}$-extensions which are best/optimal solutions w.r.t. $\mathcal{W}_2$ is selected, and so on
- If $n = 1$ then SWAFs coincide with standard WAFs

Linearly-ordered WAFs

- A particular form of SWAFs are the ones where every stratum is a singleton, that we called *Linearly ordered WAFs (LWAF)*
- Observe that for linearly ordered SWAFs (LWAFs), $CA_{\mathtt{ms}\mathcal{S}} = CA_{\mathtt{mc}\mathcal{S}}$ and $SA_{\mathtt{ms}\mathcal{S}} = SA_{\mathtt{mc}\mathcal{S}}$.

Framework

	AF		CAF		WAF			LWAF
$\mathcal{S}$	$CA_\mathcal{S}$	$SA_\mathcal{S}$	$CA_\mathcal{S}$	$SA_\mathcal{S}$	$CA_{\mathtt{ms}\mathcal{S}}$	$SA_{\mathtt{ms}\mathcal{S}}$	$CA_{\mathtt{mc}\mathcal{S}}/SA_{\mathtt{mc}\mathcal{S}}$	$CA_\mathcal{S}/SA_\mathcal{S}$
co	*NP*-c	*P*	*NP*-c	co*NP*-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c	Δ_2^P-c
st	*NP*-c	co*NP*-c	*NP*-c	co*NP*-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c	Δ_2^P-c
pr	*NP*-c	Π_2^P-c	*NP*-h, Σ_2^P	Π_2^P-c	Σ_2^P-h, Σ_3^P	Π_3^P-c	$\Delta_3^P[\log n]$-c	Δ_3^P-c
sst	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c	Σ_3^P-c	Π_3^P-c	$\Delta_3^P[\log n]$-c	Δ_3^P-c

Negative Constraints

A constraint of the form $\varphi \Rightarrow \mathtt{f}$ where φ is a conjunction containing arguments or negated arguments is called *denial* (or *negative) constraint*.

An NCAF (resp. NWAF) is a CAF (resp. WAF) where weak and strong constraints are defined by denials.

Framework

	AF		CAF		WAF			LWAF	NCAF	NWAF
$\mathcal{S}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$	$CA_{\mathcal{S}}$	$SA_{\mathcal{S}}$	$CA_{\mathtt{ms}\mathcal{S}}$	$SA_{\mathtt{ms}\mathcal{S}}$	$CA_{\mathtt{mc}\mathcal{S}}$ / $SA_{\mathtt{mc}\mathcal{S}}$	$CA_{\mathcal{S}}$ / $SA_{\mathcal{S}}$	$CA_{\mathcal{S}}$	$CA_{\mathtt{ms}\mathcal{S}}$
co	NP-c	P	NP-c	coNP-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c	Δ_2^P-c	NP-c	Σ_2^P-c
st	NP-c	coNP-c	NP-c	coNP-c	Σ_2^P-c	Π_2^P-c	$\Delta_2^P[\log n]$-c	Δ_2^P-c	NP-c	Σ_2^P-c
pr	NP-c	Π_2^P-c	NP-h, Σ_2^P	Π_2^P-c	Σ_2^P-h, Σ_3^P	Π_3^P-c	$\Delta_3^P[\log n]$-c	Δ_3^P-c	NP-c	Σ_2^P-c
sst	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c	Σ_3^P-c	Π_3^P-c	$\Delta_3^P[\log n]$-c	Δ_3^P-c	Σ_2^P-c	Σ_3^P-c

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Conclusions and future work

- We have introduced a general argumentation framework where both strong and weak constraints can be easily expressed
- Weak constraints allow for selecting best or optimal extensions satisfying some conditions on arguments, if possible
- Our complexity analysis shows how the several forms of constrains impact on the complexity of credulous and skeptical reasoning
- Constraints, especially weak ones, generally increase the expressivity of AFs

FW) Considering more general forms of constraints, not only using variables ranging on the sets of arguments, but also constraints allowing to express conditions on aggregates (e.g., at least n arguments from a given set S should be accepted/rejected)

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FW) Considering more general forms of constraints, not only using variables ranging on the sets of arguments, but also constraints allowing to express conditions on aggregates (e.g., at least n arguments from a given set S should be accepted/rejected)

Thank you!

... any question?