# Count Queries in Probabilistic Spatio-Temporal Knowledge Bases with Capacity Constraints

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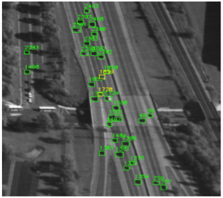
# Tracking moving objects (1/2)

- Tracking moving objects is fundamental in several application contexts (e.g. environment protection, product traceability, traffic monitoring, mobile tourist guides, analysis of animal behavior, etc.)

http://www.merl.com/publications/TR2008-010


http://www.edimax.com/au/



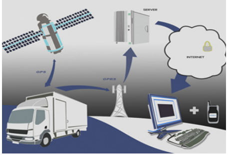
http://iris.usc.edu/people/medioni/current_research.html


http://www.i3b.org/content/wildlife-behavior


http://www.science20.com/news_articles/german_research_center_artificial_intelligence_smart_eye_tracking_glass

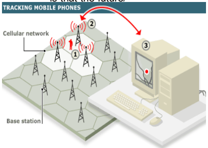
# Tracking moving objects (2/2)

- Location estimation techniques have limited accuracy and precision
  - limitations of technologies used (e.g. GPS, Cellular networks, WiFi, Bluetooth, RFID, etc.)
  - limitations of the estimation methods (e.g., proximity to antennas, triangulation, signal strength sample map, dead reckoning, etc.)

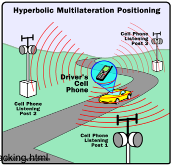


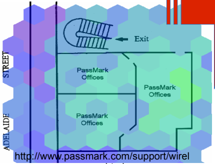
http://www.nitrobahn.com/conceptz/self-driving-cars-is-that-the-future/

http://www.ayantra.com/traffic-control-monitoring.html




http://www.gksoft.in/2014/07/mobile-phone-tracking.html




http://www.passmark.com/support/wireless_coverage_map.html

object inside a region at a time with uncertain probability

# SPOT framework

- SPOT: a declarative framework for the representation and processing of probabilistic spatio-temporal data with uncertain probabilities [Parker, Subrahmanian, Grant. TKDE '07]
- A SPOT database is a set of atoms $loc(id, r, t)[\ell, u]$
- $loc(id, r, t)[\ell, u]$ means that "object $id$ is/was/will be inside region $r$ at time $t$ with probability in the interval $[\ell, u]$".

## Example

$loc(id_1, r_7, 0)[.9, 1]$
$loc(id_1, r_8, 1)[.6, .8]$
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$loc(id_2, r_5, 1)[.4, .8]$
$loc(id_2, r_2, 2)[.4, .6]$
$loc(id_2, r_1, 2)[.3, .6]$
$loc(id_3, r_7, 0)[.9, 1]$
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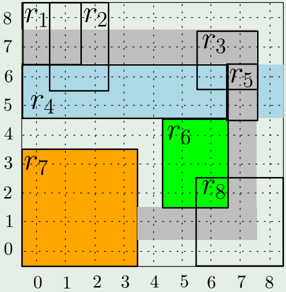


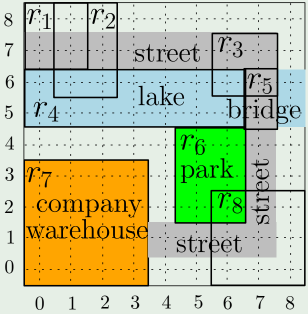

| Atoms' region | bottom-left endpoint | top-right endpoint |
|---|---|---|
| $r_1$ | (0, 7) | (1, 8) |
| $r_2$ | (1, 6) | (2, 8) |
| $r_3$ | (6, 6) | (7, 7) |
| $r_4$ | (0, 5) | (6, 6) |
| $r_5$ | (7, 5) | (7, 6) |
| $r_6$ | (5, 2) | (6, 4) |
| $r_7$ | (0, 0) | (3, 3) |
| $r_8$ | (6, 0) | (8, 2) |

# Limits of SPOT DBs

- Although PST atoms express much useful information, they **cannot express** additional knowledge such as constraints on how many objects are allowed in a region, i.e., ***capacity constraints***

## Example

1) There cannot be more than one truck on the bridge (region $r_5$) at any time
2) The number of trucks in the company warehouse is between 1 and 3 at any time between 0 and 1
3) No truck can be in the lake or the botanic park at any time point



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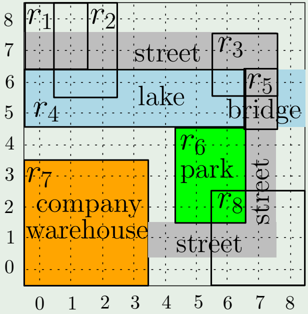

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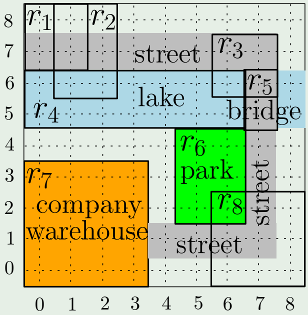

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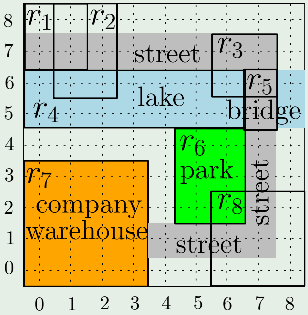

# Probabilistic spatio-temporal KBs with capacity constraints

- We introduce probabilistic spatio-temporal (PST) knowledgebases (KB) consisting of

1) atomic statements, such as those representable in the SPOT framework
2) *capacity constraints*, each of them expressing lower- and/or upper-bounds on the number of objects that can be in a certain region.

- Formal semantics, in terms of worlds, interpretations, and models
- Complexity of checking consistency of PST KBs
  - NP-complete in general
  - Restricted classes of PST KBs for which the problem is in PTIME
- Count queries over (consistent) PST KBs:
  "How many objects are inside region $q$ at time $t$?"
  - Formal semantics
  - Complexity
  - Show how checking consistency can be exploited for query answering

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# Outline

# PST atoms

- We assume a finite set *ID* of *object ids*, a finite set *Space* of *spatial points*.
- A non-empty subset of *Space* is called a *region*.
- Arbitrarily large but fixed size window of time $T = [0, 1, \ldots, tmax]$.

A *spatio-temporal atom* (*st-atom*) is an expression of the form $loc(id, r, t)$, where $id \in ID$, $\emptyset \subsetneq r \subseteq Space$, and $t \in T$.

**Definition (PST atom – SPOT atom in the previous framework)**

A PST *atom* is an st-atom $loc(id, r, t)$ annotated with a probability interval $[\ell, u] \subseteq [0, 1]$ – denoted as $loc(id, r, t)[\ell, u]$.

- $loc(id, r, t)[\ell, u]$ says that object *id* is/was/will be inside region *r* at time *t* with probability in the interval $[\ell, u]$
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# Capacity Constraints

## Definition (Capacity constraint)

A *capacity constraint* is an expression of the form $capacity(r, k_1, k_2, t)$, where $r$ is a region, $k_1$ and $k_2$ are two integers such that $0 \leq k_1 \leq k_2 \leq |ID|$, and $t$ is a time point in $T$.

## Example

1) $\kappa_{1,t} = capacity(r_5, 0, 1, t)$ with $t \in [0, 2]$,
there cannot be more than one truck on the bridge (region $r_5$) at any time between 0 and 2
2) $\kappa_{2,t} = capacity(r_7, 1, 3, t)$, with $t \in [0, 1]$,
the number of trucks in the company warehouse (region $r_7$) is between 1 and 3 at any time between 0 and 1
3) $\kappa_{3,t} = capacity(r_4, 0, 0, t)$ and
$\kappa_{4,t} = capacity(r_6, 0, 0, t)$, with $t \in [0, 2]$,
no truck can be in the lake (region $r_4$) or the botanic park (region $r_6$) at any time point (assuming $tmax = 2$)

# PST knowledge base

## Definition (PST knowledge base)

A PST knowledge base is a pair $\langle \mathcal{A}, \mathcal{C} \rangle$, where $\mathcal{A}$ is a finite set of PST atoms and $\mathcal{C}$ is a finite set of capacity constraints.

## Example

$loc(id_1, r_7, 0)[.9, 1]$
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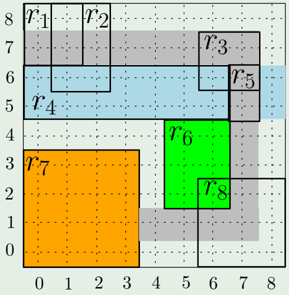


$\kappa_{1,t} = capacity(r_5, 0, 1, t)$
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# World

- A world specifies a possible trajectory for each object $id \in ID$ (i.e., says where in *Space* object $id$ was/is/will be at each time $t \in T$)

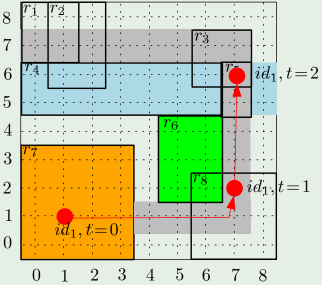
## Definition (World)

A world $w$ is a function, $w : ID \times T \to Space$

## Example

World $w_1$ describing the positions of $id_1$, $id_2$ and $id_3$ for time points in $[0, 2]$:

$$w_1(id_1, 0) = (1, 1)$$
$$w_1(id_1, 1) = (7, 2)$$
$$w_1(id_1, 2) = (7, 6)$$



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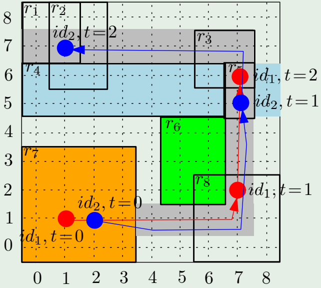
$$w_1(id_2, 0) = (2, 1)$$
$$w_1(id_2, 1) = (7, 5)$$
$$w_1(id_2, 2) = (1, 7)$$



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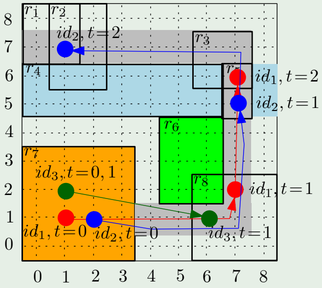

# Satisfaction

## Definition (Satisfaction)

A world $w$ *satisfies* an st-atom $a = loc(id, r, t)$, denoted $w \models a$, iff $w(id, t) \in r$. Moreover, $w$ *satisfies* a capacity constraint $\kappa = capacity(r, k_1, k_2, t)$, denoted $w \models \kappa$, iff $k_1 \leq |\{id \in ID(\mathcal{K}) \mid w(id, t) \in r\}| \leq k_2$.

## Example

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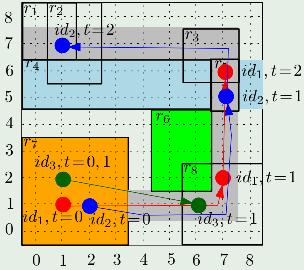



$w_1 \models loc(id_1, r_7, 0)$,
as $w_1(id_1, 0) = (1, 1) \in r_7$

$\forall t \in [0, 2],\ w_1 \models capacity(r_5, 0, 1, t)$
as $\{id \in ID(\mathcal{K}) \mid w_1(id, 0) \in r_5\} = \emptyset$
$\{id \in ID(\mathcal{K}) \mid w_1(id, 1) \in r_5\} = \{id_2\}$
$\{id \in ID(\mathcal{K}) \mid w_1(id, 2) \in r_5\} = \{id_1\}$

# Interpretations

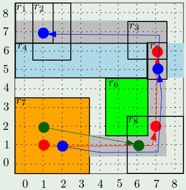
## Definition (Interpretation)

An *interpretation* $I$ for $\mathcal{K}$ is a PDF over the set $\mathcal{W}(\mathcal{K})$ of all worlds of $\mathcal{K}$.

- $I(w)$ is the probability that $w$ describes the actual trajectories of all objects

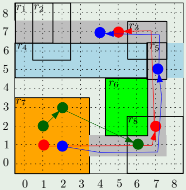
## Example (Interpretation $I$)

$w_1$



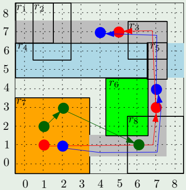

$I(w_1) = 0.6$

$w_2$



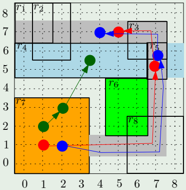

$I(w_2) = 0.2$

$w_3$




$I(w_3) = 0.2$

$w_4$




$I(w_4) = 0$

and all other words are assigned probability equal to zero by interpretation $I$

- Only the interpretations that are compatible with the information in $\mathcal{K}$ (PST atoms + Capacity constraints) are models

# Models

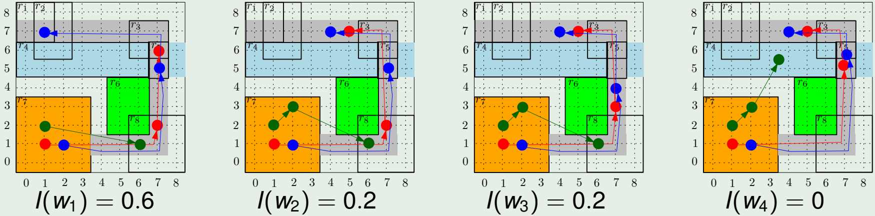
## Definition (Model)

A model $M$ for $\mathcal{K} = \langle \mathcal{A}, \mathcal{C} \rangle$ is an interpretation for $\mathcal{K}$ such that:

(i) $\forall\ loc(id, r, t)[\ell, u] \in \mathcal{A}, \left( \sum_{w | w \models loc(id,r,t)} M(w) \right) \in [\ell, u];$

(ii) $\forall\ \kappa \in \mathcal{C}, \sum_{w | w \not\models \kappa} M(w) = 0.$

## Example (Model $M$)




- For atom $loc(id_1, r_7, 0)[.9, 1]$,
  $\sum_{w | w \models loc(id_1, r_7, 0)} M(w) = M(w_1) + M(w_2) + M(w_3) = 1 \in [.9, .1]$

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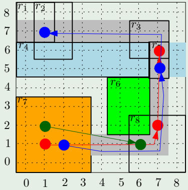
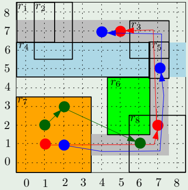
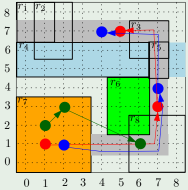


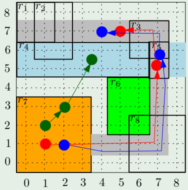
$I(w_1) = 0.6$



$I(w_2) = 0.2$



$I(w_3) = 0.2$



$I(w_4) = 0$

- $M(w_4) = 0$ since $w_4$ violates the constraint $\kappa_{1,1} = capacity(r_5, 0, 1, t)$, as there are 2 trucks on the bridge at time 1 according $w_4$

# Consistency

- The set of models for $\mathcal{K}$ will be denoted as $\mathbf{M}(\mathcal{K})$.
- $\mathcal{K}$ is *consistent* iff there exists a model for it (i.e., $\mathbf{M}(\mathcal{K}) \neq \emptyset$)
- PST KB of our running example is consistent

# Outline

# Complexity

## Theorem

*Deciding whether a* PST *KB* $\mathcal{K}$ *is consistent is NP-complete.*

- Membership: deciding whether $\mathcal{K}$ is consistent corresponds to checking the feasibility of

$$LP(\mathcal{K}) := \begin{cases} (1) & \forall\ loc(id,r,t)[\ell,u] \in \mathcal{A}, \\ & \quad (a)\ \ \ell \le \sum_{w_i \mid w_i \models loc(id,r,t)} v_i \\ & \quad (b)\ \ \sum_{w_i \mid w_i \models loc(id,r,t)} v_i \le u \\ (2) & \forall \kappa \in \mathcal{C}, \sum_{w_i \mid w_i \not\models \kappa} v_i = 0 \\ (3) & \sum_{w_i \mid w_i \in \mathcal{W}(\mathcal{K})} v_i = 1 \\ (4) & \forall w_i \in \mathcal{W}(\mathcal{K}),\ v_i \ge 0 \end{cases}$$

- $v_i$ represents probability $M(w_i)$ assigned to $w_i \in \mathcal{W}(\mathcal{K})$ by $M \in \mathbf{M}(\mathcal{K})$
- Exponential number of variables $v_i$ (i.e., $|\mathcal{W}(\mathcal{K})| = |Space|^{|ID|\cdot|T|}$)

# Complexity

## Theorem

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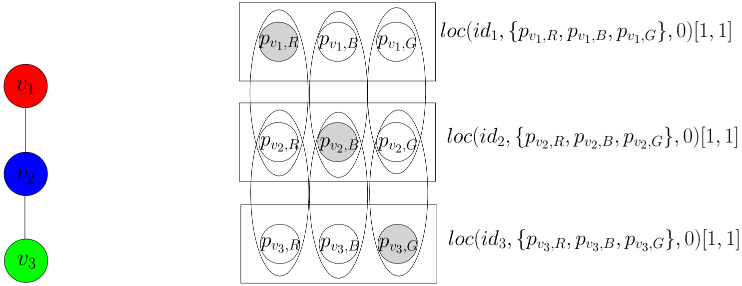
# Membership in NP

- It can be shown that $LP(\mathcal{K})$ is feasible iff there is a solution for $LP(\mathcal{K})$ consisting of at most $2 \cdot |\mathcal{A}| + |\mathcal{C}| + 1$ non-zero variables (it follows from a well-known result on the size of solutions of linear programming problems [Papadimitriou, Steiglitz '82])
- Guess an assignment $s'$ consisting of $2 \cdot |\mathcal{A}| + |\mathcal{C}| + 1$ pairs $\langle v_i, \text{value of } v_i \rangle$,
- Check in polynomial time whether $s'$ is a solution of $LP^*(\mathcal{K})$, obtained from $LP(\mathcal{K})$ by keeping in it only the variables in $s'$
- If $s'$ is a solution of $LP^*(\mathcal{K})$, then $LP(\mathcal{K})$ is feasible

# NP-hardness

- Reduction from 3-COLORING problem
- Given $G = \langle V, E \rangle$, use 3 points $p_{v,R}$, $p_{v,G}$,$p_{v,B}$ in *Space* for each $v \in V$
- PST atom $loc(id_v, \{p_{v,R}, p_{v,G}, p_{v,B}\}, 0)[1,1]$ for each vertex $v \in V$
- $capacity(\{p_{i,col}, p_{j,col}\}, 0, 1, 0)$ for each edge $(i, j) \in E$ and color $col \in \{R, G, B\}$




- $G$ is 3-colorable iff $\mathcal{K}$ is consistent

# Tractable cases

- Capacity constraints allowing no objects in some regions (e.g., there cannot be trucks in the lake)

**Theorem**

*Let $\mathcal{K} = \langle \mathcal{A}, \mathcal{C} \rangle$ be a* PST *KB. If $\mathcal{C}$ consists of capacity constraints of the form $capacity(r, 0, 0, t)$, then checking whether $\mathcal{K}$ is consistent is in PTIME.*

- Proof hint: it can be reduced to checking consistency of a KB having no capacity constraints, which is in PTIME [Parker, Subrahmanian, Grant. TKDE '07]
- $capacity(r, 0, 0, t)$ can be translated into the set of additional atoms $\forall id \in ID,\ loc(id, Space \setminus r, t)[1, 1]$

# Sufficient conditions for checking consistency (1/2)

- Upper bounds of all PST atoms is 1 and
- regions in different capacity constraints are disjoint

**Theorem**

*Let* $\mathcal{K} = \langle \mathcal{A}, \mathcal{C} \rangle$ *be a* PST *KB that satisfies the following conditions:*

- $\mathcal{A}$ *consists of* PST *atoms of the form* $loc(id, r, t)[\ell, 1]$ *and there are no two distinct* PST *atoms in* $\mathcal{A}$ *for the same object id and time point t, and*
- *for every time point t, every pair of distinct capacity constraints* $capacity(r, k_1, k_2, t)$ *and* $capacity(r', k_1', k_2', t)$ *in* $\mathcal{C}$ *is such that* $r \cap r' = \emptyset$.

*Deciding if there exists a world* $w \in \mathcal{W}(\mathcal{K})$ *s.t. (i)* $w \models \mathcal{C}$ *and (ii)* $w(id, t) \in r$ *for every* $loc(id, r, t)[\ell, 1]$ *in* $\mathcal{A}$ *with* $\ell > 0$*, is in PTIME. If such a world exists, then* $\mathcal{K}$ *is consistent.*

- reduction to the problem of deciding if a flow network admits a feasible circulation

# Sufficient conditions for checking consistency (2/2)

- A PST KB $\langle \mathcal{A}, \mathcal{C} \rangle$ is called *simple* iff for every time point $t \in T$, there is at most one capacity constraint of the form $capacity(r, k_1, k_2, t)$ in $\mathcal{C}$

## Theorem

*Let $\mathcal{K} = \langle \mathcal{A}, \mathcal{C} \rangle$ be a simple* PST *KB. If $\langle \mathcal{A}, \emptyset \rangle$ is consistent and, for every $capacity(r, k_1, k_2, t) \in \mathcal{C}$, $[z, Z] \subseteq [k_1, k_2]$, where*

$$z = \min_{M \in \mathbf{M}(\langle \mathcal{A}, \emptyset \rangle)} |\{id \mid id \in ID \wedge \left( \sum_{w | w(id,t) \in r} M(w) \right) = 1\}|,$$

$$Z = \max_{M \in \mathbf{M}(\langle \mathcal{A}, \emptyset \rangle)} |\{id \mid id \in ID \wedge \left( \sum_{w | w(id,t) \in r} M(w) \right) \neq 0\}|,$$

*then $\mathcal{K}$ is consistent. Checking consistency under such conditions is in PTIME.*

- Computing $[z, Z]$ is in PTIME [Grant, Molinaro, Parisi. SUM 2013]

# Outline

# Syntax and semantics

- $Count(q, t)$ asks “How many objects are inside region $q$ at time $t$?”
- Ranking answer: set of pairs $\langle i, [\ell_i, u_i]\rangle$ where
  - $i$ is the number of objects that may be in $q$ at time $t$
  - $\ell_i$ and $u_i$ are the minimum and maximum probabilities of having exactly $i$ objects in $q$ at a time $t$ over all models
- For a given model $M$, the probability of having exactly $i$ objects in a region $q$ at a time point $t$ w.r.t. $M$ is $Prob_M(q, i, t) = \sum_{w \mid w \models capacity(q,i,t)} M(w)$

## Definition (Ranking Answer)

The ranking answer to a count query $Q = Count(q, t)$ w.r.t. $\mathcal{K}$ is:

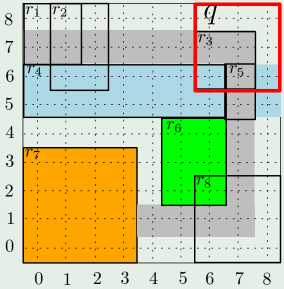
$$Q(\mathcal{K}) = \{\langle i, [\ell_i, u_i]\rangle \mid \quad 0 \leq i \leq |ID| \wedge \quad \ell_i = \min_{M \in \mathbf{M}(\mathcal{K})} Prob_M(q, i, t) \wedge \\ u_i = \max_{M \in \mathbf{M}(\mathcal{K})} Prob_M(q, i, t)\}.$$

# Example

## Example

- How many trucks are in $q$ (the red square) at time 2?

$loc(id_1, r_7, 0)[.9, 1]$
$loc(id_1, r_8, 1)[.6, .8]$
$loc(id_1, r_3, 2)[.4, .6]$
$loc(id_2, r_7, 0)[.9, 1]$
$loc(id_2, r_5, 1)[.4, .8]$
$loc(id_2, r_2, 2)[.4, .6]$
$loc(id_2, r_1, 2)[.3, .6]$
$loc(id_3, r_7, 0)[.9, 1]$
$loc(id_3, r_7, 1)[.9, 1]$




$\kappa_{1,t} = capacity(r_5, 0, 1, t)$
$t \in [0, 2]$
$\kappa_{2,t} = capacity(r_7, 1, 3, t)$,
$t \in [0, 1]$,
$\kappa_{3,t} = capacity(r_4, 0, 0, t)$ ,
$\kappa_{4,t} = capacity(r_6, 0, 0, t)$,
$t \in [0, 2]$,

- Ranking answer $Q(\mathcal{K}) = \{\langle 0, [.4, .6]\rangle, \langle 1, [.4, 1]\rangle, \langle 2, [0, .3]\rangle, \langle 3, [0, .1]\rangle\}$
- For instance, $\langle 1, [.4, 1]\rangle$ says that the probability of having exactly one object in $q$ at time 2 is between .4 and 1.

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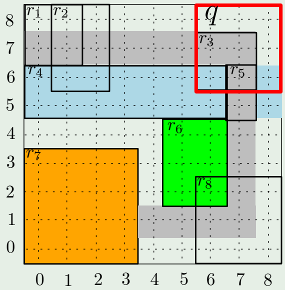


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- For instance, $\langle 1, [.4, 1]\rangle$ says that the probability of having exactly one object in $q$ at time 2 is between .4 and 1.

# Complexity

**Theorem**

*Computing $Q(\mathcal{K})$ is $FP^{NP[\log n]}$-hard.*

- Reduction to our problem from the $FP^{NP[\log n]}$-hard problem CLIQUE SIZE: determine the size $\sigma$ of the largest clique of a graph $G = \langle V, E \rangle$
- Proof hint: An id $id_v$ and two spatial points $p_{v,in}, p_{v,out}$ for each $v \in V$
- PST atom saying that $id_v$ must be at one of the two points $p_{v,in}, p_{v,out}$
- $capacity(\{p_{i,in}, p_{j,in}\}, 0, 1, 0)$ for each $(i, j) \in (V \times V) \setminus E$ saying that no more than one object can be in the region consisting of two *in* points associated with a pair of vertices *not* connected by an edge
- $Q = Count(\{p_{1,in}, \ldots, p_{n,in}\}, 0)$.
- The size of the largest clique of $G$ is $\sigma$ iff $Q(\mathcal{K}) = \{\langle i, [0, 1]\rangle \mid 0 \le i \le \sigma\} \cup \{\langle i, [0, 0]\rangle \mid \sigma < i \le |ID|\}$.

# Using consistency checking to answering queries

- Solving some instances of the consistency check problem allows us to answer some count queries
- Given $\mathcal{K} = \langle \mathcal{A}, \mathcal{C} \rangle$, we check consistency of $\mathcal{K}' = \langle \mathcal{A}, \mathcal{C}' \rangle$ to get the answers

## Proposition

*Let $Q = Count(q, t)$ and $\mathcal{K} = \langle \mathcal{A}, \mathcal{C} \rangle$.*

- *If $\mathcal{K}' = \langle \mathcal{A}, \mathcal{C} \cup \{capacity(q, k_1, k_2, t)\} \rangle$ is consistent, then $\ell_i = 0$ in $Q(\mathcal{K})$ for all $i$ such that $i < k_1$ or $i > k_2$.*
- *If $\mathcal{K}' = \langle \mathcal{A}, \mathcal{C} \cup \{capacity(Space \setminus q, k_1, k_2, t)\} \rangle$ is consistent, then $u_i = 1$ in $Q(\mathcal{K})$ for all $i \in [|ID| - k_2, |ID| - k_1]$.*

# Outline

# Conclusions and future work

- A declarative language suitable in many applications dealing with uncertain spatio-temporal data
- Capacity constraints allow us to model semantic information commonly arising in practice
- We have investigated the complexity of checking consistency and answering count queries
- Intractable in general, but tractable approaches for restricted cases
- Further issues that we plan to investigate:
  - other tractable cases
  - the interaction between capacity constraints and the universal denial constraints proposed in [Parisi, Grant JAIR 2016] to get a unified approach that allows for a wide range of constraints to be expressed
  - the problems of repairing and querying inconsistent PST KBs with capacity constraints (following [Parisi, Grant IJAR 2017] where the problem of restoring consistency of PST KBs without integrity constraints has been explored)

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Thank you!

... any question?

# Location estimation techniques

- Location estimation techniques build on different technologies (e.g. GPS, Cellular networks, WLAN, Bluetooth, RFID, etc.)
  - proximity techniques derive the location of an object w.r.t. its vicinity to antennas
  - triangulation uses the triangle geometry to compute locations of an object.
  - scene analysis techniques (e.g. fingerprinting technique) involve examination and matching a video/image or electromagnetic characteristics viewed/sensed from an object
  - Dead reckoning techniques provide estimation of the location of an object based on the last known position, assuming that the direction of motion and either the velocity of the target object or the travelled distance are known
  - hybrid techniques
- Several sources of spatial temporal information (e.g. GPS, Cellular networks, WLAN, Wi-Fi), Bluetooth, Zigbee, Ultra-wideband (UWB), and Radio-frequency identification (RFID), or infrared (IR)

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