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An Incremental Approach to Structured Argumentation over Dynamic Knowledge Bases

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Dynamic Structured Argumentation

- Argumentation frameworks are often dynamic (change over time) as a consequence of the fact that argumentation is inherently dynamic (change mind/opinion, new available knowledge)

- We focus on Defeasible Logic Programming, a formalism that combines results of Logic Programming and Defeasible Argumentation.

- We devise an incremental technique for computing conclusions in structured argumentation frameworks (avoiding wasted effort due to recomputation from scratch)
Defeasible Logic Programming

A DeLP Program

DeLP considers two kinds of program rules:

- **Defeasible** rules to represent tentative information, and
- **Strict** rules used to represent strict knowledge.

**Example (A DeLP-program $\mathcal{P}_1$)**

Consider the DeLP-program $\mathcal{P}_1 = (\Pi_1, \Delta_1)$, where:

$$\Pi_1 = \{ \neg a, t, b, (d \leftarrow t) \}$$

$$\Delta_1 = \begin{cases} (i \leftarrow s), & (s \leftarrow h), & (h \leftarrow b), \\ (\neg h \leftarrow d, t), & (\neg i \leftarrow \neg a, s), & (a \leftarrow t), \\ (s \leftarrow d), & (h \leftarrow d), & (\neg f \leftarrow \neg e), \\ (\neg e \leftarrow \neg h, \neg a) \end{cases}$$
Defeasible Logic Programming

**Argument**

Given a DeLP program \( \mathcal{P} = (\Pi, \Delta) \) and a literal \( \alpha \), we say that \( \langle A, \alpha \rangle \) is an argument for \( \alpha \) if \( A \) is a set of defeasible rules of \( \Delta \) such that:

1. there is a derivation for \( \alpha \) from \( \Pi \cup A \),
2. the set \( \Pi \cup A \) is not contradictory, and
3. \( A \) is minimal (i.e., there is no proper subset \( A' \) of \( A \) satisfying both (i) and (ii)).

**Example (An argument for \( \mathcal{P}_1 \))**

\[
\langle A_1, i \rangle = \langle \{(i \leftarrow s), (s \leftarrow h), (h \leftarrow b)\}, i \rangle
\]
Argumentation Line

- A sequence of arguments obtained from a DeLP program, where each element of the sequence is a defeater of its predecessor.

**Example (Argumentation Line)**

\[ A_0, A_1, A_2, A_3 \]

**Example (An argumentation line for \( P_1 \))**

Given the two arguments:  
\[ \langle A_1, i \rangle = \langle \{(i \rightarrow s), (s \rightarrow h), (h \rightarrow b)\}, i \rangle, \text{ and} \]
\[ \langle A_2, \sim i \rangle = \langle \{\sim i \leftarrow \sim a, s), (s \leftarrow d)\}, \sim i \rangle \]

an argumentation line is the following: \([A_1, A_2]\)
Given an argument $A$ for a literal $L$, the dialectical tree contains all acceptable argumentation lines that start with that argument.

It allows to determine the status for a given argument.
All leaves are marked as **Undefeated**.

An argument in the tree is marked as **Defeated** if and only if it has at least a child marked as **Undefeated**.
Status of literals

The status of literals allow us to determine the conclusions we can draw from a DeLP-program. 

\( S_P : \text{Lit} \rightarrow \{ \text{IN, OUT, UNDECIDED, UNKNOWN} \} \) assigning a status to each literal w.r.t. \( P \) as follows:

- \( S_P(\alpha) = \text{IN} \) if there exists a (marked) dialectical tree whose root \( \alpha \) is *Undefeated*
- \( S_P(\alpha) = \text{OUT} \) if \( S_P(\sim\alpha) = \text{IN} \)
- \( S_P(\alpha) = \text{UNDECIDED} \) if neither \( S_P(\alpha) = \text{IN} \) nor \( S_P(\alpha) = \text{OUT} \)
- \( S_P(\alpha) = \text{UNKNOWN} \) if \( \alpha \notin \text{Lit}_P \), i.e., \( \alpha \) is not in the language of the program

**Example (Arguments from the previous program)**

Given \( P \), then \( S_{P_1}(h) = \text{IN} \), \( S_{P_1}(a) = \text{OUT} \), and \( S_{P_1}(i) = \text{UNDECIDED} \).
Theorem  Given $\mathcal{P}$ and a literal $\alpha \in \text{Lit}_\mathcal{P}$, deciding whether there is an argument for $\alpha$ w.r.t. $\mathcal{P}$ is NP-complete.

Corollary  Let $\mathcal{P} = (\Pi, \Delta)$ be a DeLP-program such that for all $r \in (\Pi \cup \Delta)$, $|\text{body}(r)| \leq 2$. Deciding whether there is an argument for $\alpha \in \text{Lit}_\mathcal{P}$ w.r.t. $\mathcal{P}$ is NP-complete.

Proposition  Given $\mathcal{P} = (\Pi, \Delta)$ and a literal $\alpha \in \text{Lit}_\mathcal{P}$, deciding whether there is an argument for $\alpha$ w.r.t. $\mathcal{P}$ is in PTIME if either (i) $\alpha$ does not depend in $G(\mathcal{P})$ on literals $\beta$ and $\gamma$ such that $\{\beta, \gamma\} \cup \Pi$ is contradictory, or (ii) $\alpha$ is not in $G(\mathcal{P})$.

Corollary  Let $\mathcal{P} = (\Pi, \Delta)$ be a DeLP-program such that for all $r \in (\Pi \cup \Delta)$, $|\text{body}(r)| \leq 2$. Deciding whether $S_\mathcal{P}(\alpha) = \text{IN}$, $S_\mathcal{P}(\alpha) = \text{OUT}$, or $S_\mathcal{P}(\alpha) = \text{UNDETERMINED}$, for $\alpha \in \text{Lit}_\mathcal{P}$ is NP-hard.
Updates

An update consists of modifying a DeLP-program $\mathcal{P}$ into a new DeLP-program $\mathcal{P}'$ by adding or removing a strict or a defeasible rule.

Example (Perform $u = + (\sim i \prec h)$ on $\mathcal{P}_1$)

The updated DeLP-program $\mathcal{P}'_1 = (\Pi'_1, \Delta'_1)$, is as follows:

$\Pi'_1 = \Pi_1 = \{\sim a, t, b, (d \leftarrow t)\}$

$\Delta'_1 = \Delta_1 = \{(i \prec s), (s \prec h), (h \prec b),\}
\{(\sim h \prec d, t), (\sim i \prec \sim a, s), (a \prec t),\}
\{(s \prec d), (h \prec d), (\sim f \prec \sim e),\}
\{\sim e \prec \sim h, \sim a\}
\cup \{\sim i \prec h\}$

If $r$ is a strict rule and $u = + r$, then $\mathcal{P}' = ((\Pi \cup \{r\}), \Delta)$ if $(\Pi \cup \{r\})$ is guaranteed to be not contradictory, otherwise $\mathcal{P}' = \mathcal{P}$. 
After performing an update the conclusion that can be derived may change.

Should we recompute the status of literals from scratch?

The fact that computing the status of arguments is hard motivated the investigation of incremental techniques.
Overview of our incremental approach

Two main steps:

1) First, we check if the update is *irrelevant* (the status of all literals are preserved). In such a case we simply return the initial status $S_P$.

2) To efficiently deal with *relevant* updates, we identify the subset of literals whose status needs to be recomputed after performing an update, and only recompute their status.
Hyper-graph for a DeLP-Program

Given a program $\mathcal{P}$, $G(\mathcal{P}) = \langle N, H \rangle$ is defined as follows:

- If there is a strict derivation in $\Pi$ for literal $\alpha$, then $\alpha \in N$;
- For each strict rule $\alpha_0 \leftarrow \alpha_1, \ldots, \alpha_n$ (resp., defeasible rule $\alpha_0 \leftarrow^{\sim} \alpha_1, \ldots, \alpha_n$) such that $\alpha_1, \ldots, \alpha_n \in N$, then $\alpha_0 \in N$ and $(\{\alpha_1, \ldots, \alpha_n\}, \alpha_0) \in H$;
- For each pair of nodes in $N$ representing complementary literals $\alpha$ and $\sim \alpha$, both $(\{\alpha\}, \sim \alpha) \in H$ and $(\{\sim \alpha\}, \alpha) \in H$.

Example (Hyper-graph $G(\mathcal{P}_1)$ for $\mathcal{P}_1$)

Consider the DeLP-program $\mathcal{P}_1 = (\Pi_1, \Delta_1)$, where:

$\Pi_1 = \{\sim a, t, b, (d \leftarrow t)\}$

$\Delta_1 = \begin{cases} (i \leftarrow s), & (s \leftarrow h), & (h \leftarrow b), \\ (\sim h \leftarrow d, t), & (\sim i \leftarrow \sim a, s), & (a \leftarrow t), \\ (s \leftarrow d), & (h \leftarrow d), & (\sim f \leftarrow \sim e), \\ (\sim e \leftarrow \sim h, \sim a) \end{cases}$
Reachable and Preserved Literals

- We say that a node $y$ is *reachable* from a set $X$ of nodes if there exists a hyper-path from $X$ to $y$.
- We use $Reach_{G(P)}(X)$ to denote the set of all nodes that are reachable from $X$ in $G(P)$.
- $Reach_{G(P, t)}(\{d\}) = \{d, h, \sim h, s, \sim i, i, \sim e, \sim f\}$

**Lemma (1) (Preserved literals)**

Let $P$ be a DeLP-program, $u = \pm r$ an update for $P$, and $R(u, P) = Reach_{G(u, P)}(\{\text{head}(r)\})$. Let $P' = u(P)$ be the updated program, and $G(P') = \langle N', H' \rangle$ be the updated hyper-graph. Then, a literal $\alpha \in N'$ is preserved (i.e., $S_P(\alpha) = S_{P'}(\alpha)$) if $\alpha \notin R(u, P)$.

- If a literal is not reachable in the hyper-graph, then its status does not change after the update.
Irrelevant updates

Proposition (2) Status of the head of the rule

Let $\mathcal{P}$ be a DeLP-program and $r = \alpha_0 \leftarrow \alpha_1, \ldots, \alpha_n$ a defeasible rule such that \{\alpha_0, \ldots, \alpha_n\} $\subseteq$ (\text{Lit}_\mathcal{P} \cap \text{Lit}_\mathcal{P}').

1. If $S_\mathcal{P}(\alpha_0) = \text{IN}$ then $+r$ is irrelevant for $\mathcal{P}$.
2. If $S_\mathcal{P}(\alpha_0) = \text{OUT}$ then $-r$ is irrelevant for $\mathcal{P}$.

Proposition (3) Belonging to the Hyper-Graph

Let $\mathcal{P}$ be a DeLP-program and $r$ a strict rule $\alpha_0 \leftarrow \alpha_1, \ldots, \alpha_n$ or defeasible rule $\alpha_0 \leftarrow \alpha_1, \ldots, \alpha_n$ such that \{\alpha_0, \ldots, \alpha_n\} $\subseteq$ (\text{Lit}_\mathcal{P} \cap \text{Lit}_\mathcal{P}'). Update $u = \pm r$ is irrelevant for $\mathcal{P}$ if $\alpha_0$ does not belong to $G(u, \mathcal{P})$.

Proposition (4) Reachable in the Hyper-Graph

Let $\mathcal{P}$ be a DeLP-program and $r$ a strict rule $\alpha_0 \leftarrow \alpha_1, \ldots, \alpha_n$ or defeasible rule $\alpha_0 \leftarrow \alpha_1, \ldots, \alpha_n$ such that \{\alpha_0, \ldots, \alpha_n\} $\subseteq$ (\text{Lit}_\mathcal{P} \cap \text{Lit}_\mathcal{P}'). Update $u = \pm r$ is irrelevant for $\mathcal{P}$ if there is $\alpha_i$ (with $i \in [1..n]$) such that $S_\mathcal{P}(\alpha_i) = \text{OUT}$ and $\alpha_i \not\in \text{Reach}_{G(u, \mathcal{P})}(\{\alpha_0\})$. 
However, in many cases updates are not irrelevant. An update is relevant whenever it causes the status of at least one literal to change.

Example (A relevant update)

Consider again $\mathcal{P}_1$, where we have that $S_{\mathcal{P}_1}(s) = S_{\mathcal{P}_1}(t) = \text{IN}$. For update $u = +(s \leftarrow t)$, we have that $S_{u(\mathcal{P}_1)}(\sim i) = \text{IN}$, though it was UNDECIDED before performing the update. The change in the status of $s$ is caused by the new argument $\langle A_{10}, \sim i \rangle = \langle \{ (\sim i \prec \sim a, s) \}, \sim i \rangle$ for $u(\mathcal{P}_1)$ and $A_{10}$ is preferred to all the other arguments of the form $\langle A, i \rangle$.
We propose the concept of *influenced set*, which consists of the literals that are reachable in $G(u, \mathcal{P})$ from the head of the rule $r$ in the update $u$ by using only the hyper-edges whose body does not contain an unreachable literal whose status is OUT.

**Example (Influenced literals)**

Consider the update $u = +(a \leftarrow s)$ over $\mathcal{P}_1$, which yields the DeLP-program $u(\mathcal{P}_1)$. Thus, we have:

$\mathcal{R}(u, \mathcal{P}_1) = \{a, \sim a, i, \sim i, \sim e, \sim f\}$, and

$\mathcal{R}(u, \mathcal{P}_1) \supseteq \mathcal{I}(u, \mathcal{P}_1, S_{\mathcal{P}_1}) = \{a, \sim a, i, \sim i\}$. 
Dealing with Relevant Updates

**Influenced Set**

We propose the concept of *influenced set*, which consists of the literals that are reachable in $G(u, \mathcal{P})$ from the head of the rule $r$ in the update $u$ by using only the hyper-edges whose body does not contain an unreachable literal whose status is OUT.

**Example (Influenced literals)**

Consider the update $u = +(a \leftarrow s)$ over $\mathcal{P}_1$, which yields the DeLP-program $u(\mathcal{P}_1)$. Thus, we have:

$\mathcal{R}(u, \mathcal{P}_1) = \{a, \neg a, i, \neg i, \neg e, \neg f\}$, and

$\mathcal{R}(u, \mathcal{P}_1) \supseteq I(u, \mathcal{P}_1, S_{\mathcal{P}_1}) = \{a, \neg a, i, \neg i\}$. 

![Diagram of influenced set](image)
Dealing with Relevant Updates

Influenced Set: Definition

We propose the concept of *influenced set*, which consists of the literals that are reachable in $G(u, P)$ from the head of the rule $r$ in the update $u$ by using only the hyper-edges whose body does not contain an unreachable literal whose status is OUT.

**Definition (Influenced Set)**

Let $P$ be a DeLP-program, $u = \pm r$, and $S_P$ the status of literals w.r.t. $P$, and $G(u, P) = \langle N^u, H^u \rangle$.

- $I_0(u, P, S_P) = \begin{cases} \emptyset & \text{if } u \text{ is irrelevant for } P \\ \{\text{head}(r)\} & \text{otherwise} \end{cases}$

- $I_{i+1}(u, P, S_P) = I_i(u, P, S_P) \cup \{\sim \alpha \mid \exists \{\alpha\}, \sim \alpha \in H^u \text{ s.t. } \alpha \in I_i(u, P, S_P)\} \cup \{y \mid \exists (X, \alpha) \in H^u \text{ s.t. } X \cap I_i(u, P, S_P) \neq \emptyset \land X \cap OUT(u, P, S_P) = \emptyset\}$

The *influenced set* for $u$ w.r.t. $P$ and $S_P$ is then defined as $I(u, P, S_P) = I_n(u, P, S_P)$ such that $I_n(u, P, S_P) = I_{n+1}(u, P, S_P)$. 
Dealing with Relevant Updates

Inferable and Core Literals

**[Inferable]** The status of a literal for which there is no argument in the (updated) program may depend only on the status of its complementary literal—we call such literals *inferable*.

**[Core]** The core literals for a relevant update \( u = \pm r \) w.r.t. \( P \) are those in \( \text{Lit}_P \), that are influenced but are not inferable.

Example (Inferbale and Core literals)

Consider the update \( u = -(d \leftarrow t) \) over \( P_1 \), which yields the DeLP-program \( u(P_1) \). Thus, we have:

\[
\text{Infer}(u, P_1) = \{d, \sim h, \sim e, \sim f\} \\
\text{Core}(u, P_1) = \{h, s, \sim i, i\}.
\]
Dealing with Relevant Updates

Inferable and Core Literals

**[Inferable]** The status of a literal for which there is no argument in the (updated) program may depend only on the status of its complementary literal—we call such literals *inferable*.

**[Core]** The core literals for a relevant update $u = \pm r$ w.r.t. $\mathcal{P}$ are those in $\text{Lit}_{\mathcal{P}'}$ that are influenced but are not inferable.

**Example (Inferbale and Core literals)**

Consider the update $u = -(d \leftarrow t)$ over $\mathcal{P}_1$, which yields the DeLP-program $u(\mathcal{P}_1)$. Thus, we have:

$$\text{Infer}(u, \mathcal{P}_1) = \{d, \sim h, \sim e, \sim f\}$$

$$\text{Core}(u, \mathcal{P}_1) = \{h, s, \sim i, i\}.$$
Inferable and Core Literals: Definitions

The status of a literal for which there is no argument in the (updated) program may depend only on the status of its complementary literal—we call such literals \textit{inferable}. Using the hyper-graph of updated programs, we can define inferable literals as follows.

\textbf{Definition (Set of Inferable Literals)}

Let $\mathcal{P}$ be a DeLP-program, $u = \pm r$, $\mathcal{P}' = u(\mathcal{P})$, and $G(\mathcal{P}') = \langle N', H' \rangle$. The set of inferable literals for $u$ w.r.t. $\mathcal{P}$ is $\text{Infer}(u, \mathcal{P}) = \text{Lit}_{\mathcal{P}'} \setminus N'$.

The core literals for a relevant update $u = \pm r$ w.r.t. $\mathcal{P}$ are those in $\text{Lit}_{\mathcal{P}'}$ that are influenced but are not inferable.

\textbf{Definition (Set of Core Literals)}

Let $\mathcal{P}$ be a DeLP-program, $u = \pm r$, and $S_{\mathcal{P}}$ the status of the literals of $\mathcal{P}$. The set $\text{Core}(u, \mathcal{P})$ of core literals for $u$ w.r.t. $\mathcal{P}$ is $\text{Core}(u, \mathcal{P})) = (\mathcal{I}(u, \mathcal{P}, S_{\mathcal{P}}) \setminus \text{Infer}(u, \mathcal{P})) \cap \text{Lit}_{\mathcal{P}'}$. 

Dealing with Relevant Updates

Relationships for addition

Legend:

- $PR$
- $Core(u,P)$
- $Infer(u,P)$
Dealing with Relevant Updates

Relationships for deletion

Legend:

- \( PR \)
- \( Core(u,P) \)
- \( Infer(u,P) \)
Our Technique

Incremental Algorithm

**Algorithm** Dynamic DeLP-Solver

**Input:** DeLP-program $\mathcal{P}$, Initial status $S_\mathcal{P}$, Update $u = \pm r$.

**Output:** Status $S_{\mathcal{P}'}$, w.r.t. the updated program $\mathcal{P}' = u(\mathcal{P})$.

1: if one of Propositions 2–4 holds (the update is irrelevant) then

2: return $S_\mathcal{P}$; // Nothing changes

3: if $\{\text{head}(r), \sim \text{head}(r)\} \cap \text{Lit}_\mathcal{P} = \emptyset$ then

4: return $S_\mathcal{P} \cup \{(\text{head}(r), \text{DELP-SOLVER}(\mathcal{P}', \text{head}(r)))\}$; // New fresh literal

5: Let $G(\mathcal{P}') = \langle N', H' \rangle$; // Build the Hyper-Graph

6: Let $PR = \{\alpha \in N' \setminus I(u, \mathcal{P}, S_\mathcal{P})\}$; // Preserved Literals

7: for $\alpha \in PR$ do

8: $S_{\mathcal{P}'}(\alpha) \leftarrow S_\mathcal{P}(\alpha)$; // Status Preserved

9: for $\alpha \in \text{Core}(u, \mathcal{P})$ do

10: $S_{\mathcal{P}'}(\alpha) \leftarrow \text{DELP-SOLVER}(\mathcal{P}', \alpha)$; // Status must be computed

11: for $\alpha \in \text{Infer}(u, \mathcal{P})$ do

12: if $S_{\mathcal{P}'}(\neg \alpha) = \text{IN}$

13: then $S_{\mathcal{P}'}(\alpha) \leftarrow \text{OUT}$; // Status Inferred

14: else $S_{\mathcal{P}'}(\alpha) \leftarrow \text{UNDECIDED}$; // Status Inferred

15: for $\alpha \in \text{Lit} \setminus \text{Lit}_{\mathcal{P}'}$ do

16: $S_{\mathcal{P}'}(\alpha) = \text{UNKNOWN}$; // Literal is not in the language of the updated program

17: return $S_{\mathcal{P}'}$. 
**Dataset & Methodology**

**Datasets:**
Inspired by the structure of the DeLP-program in our running example, we generated a set of 40 DeLP programs, each consisting of a number of literals in \( \{180, 220\} \), of facts in \( \{10, 20\} \), of strict rules in \( \{20, 30\} \), and a number of defeasible rules in \( \{100, 150\} \). For each program, we generated 5 different rule addition/deletion updates.

**Methodology**

**[Efficiency]** For each DeLP-program \( \mathcal{P} \) in the dataset, we compared the average running time of Algorithm 1 with that of the approach from scratch, which computes the status in the updated program by directly calling the DeLP-Solver for each literal of \( \mathcal{P} \).

**[Effectiveness]** We also measured the percentage of literals whose status needs to be recomputed over the set of literals whose status is recomputed by Algorithm 1.

\[
E(u, \mathcal{P}) = \frac{|Rec(u, \mathcal{P})|}{|Core(u, \mathcal{P}) \cup Infer(u, \mathcal{P})|}
\]
Experimental Results for addition/deletion (left/right)
Results

1) We compared our technique with the computation from scratch.

2) We performed experiments that aimed at evaluating both the efficiency and effectiveness of our approach.

3) Our incremental algorithm outperforms the computation from scratch.

4) For almost half of the updates performed, the proposed technique computes only the status of literals whose status actually needs to be recomputed.
Outline

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   • Dealing with Irrelevant Updates
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   • Our Technique

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6 Conclusions & Future Work
* We have taken the first steps in tackling the problem of avoiding wasted effort when determining the warrant status of literals in a DeLP program after that a (defeasible or strict) rule is added/removed.

* Our incremental approach outperforms the computation from scratch (especially if the average number of literals reachable from an update is less than 33%).

FW1) Further developing these techniques, as well as developing similar ones for fact addition and deletion, and the more general case of simultaneously adding or deleting a set of rules and facts.

FW2) We believe the basic ideas in the framework could carry over to other frameworks, v.g. ASPIC+, ABA.
Thank you!

... any question argument?