

# Incremental Computation of Warranted Arguments in Dynamic Defeasible Argumentation: The Rule Addition Case

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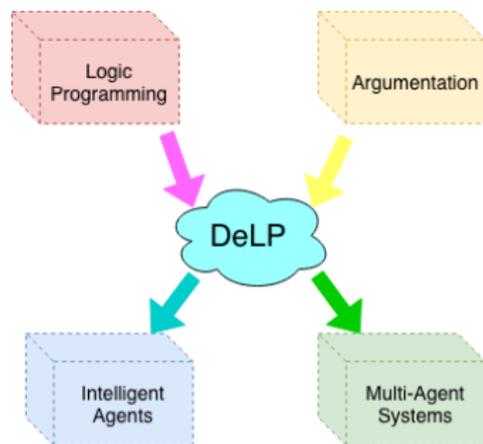
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# Dynamic Structured Argumentation

- A general way for representing arguments and relationships (defeats) between them
- It allows representing dialogues, making decisions, and handling inconsistency and uncertainty
- Several kinds of Argumentation Frameworks (e.g. Abstract Argumentation, **Structured Argumentation**)
- A well-known formalism for structured argumentation is DeLP: **D**efeasible **L**ogic **P**rogramming
- Argumentation frameworks are often dynamic (change over time) as a consequence of the fact that argumentation is inherently dynamic (change mind/opinion, new available knowledge)
- We devise an incremental technique for computing conclusions in structured argumentation frameworks (avoiding wasted effort due to recomputation from scratch)

# DeLP

- We focus on **Defeasible Logic Programming**, a formalism that combines results of Logic Programming and Defeasible Argumentation.
- DeLP is a knowledge representation language, where defeasible and non-defeasible rules can be expressed.
- The language has two different negations: classical negation, used for representing contradictory knowledge and negation as failure, used for representing incomplete information.
- A defeasible argumentation inference mechanism for warranting the conclusions that are entailed.



# Outline

- 1 Introduction
  - Motivation
- 2 Background
  - Defeasible Logic Programming
- 3 Incremental Computation
  - Updates
  - Dealing with Irrelevant Updates
  - Dealing with Relevant Updates
  - Our Technique
- 4 Implementation & Experiments
- 5 Conclusions and future work

# A DeLP Program

DeLP considers two kinds of program rules: **defeasible** rules to represent tentative information, and **strict** rules used to represent strict knowledge.

## Example

Consider the DeLP-program  $\mathcal{P}_1 = (\Pi_1, \Delta_1)$ , where:

$$\begin{aligned} \Pi_1 &= \{w, t, z, (p \leftarrow t)\} \\ \Delta_1 &= \left\{ \begin{array}{cccc} (\sim a \leftarrow y), & (y \leftarrow x), & (x \leftarrow z), & (\sim x \leftarrow p, t) \\ (a \leftarrow w, y), & (\sim w \leftarrow t), & (y \leftarrow p), & (x \leftarrow p) \end{array} \right\} \end{aligned}$$

The (non-contradictory) set of literals that can be derived from  $\Pi_1$  is  $\{w, t, z, p\}$ . However, both  $a$  and  $\sim a$  can be derived from  $\mathcal{P}_1$  using the following sets of rules and facts:

$$(a \leftarrow w, y), (y \leftarrow p), (p \leftarrow t), (t) \text{ and } (\sim a \leftarrow y), (y \leftarrow p), (p \leftarrow t), (t),$$

respectively.

# Argument

Given a DeLP program  $\mathcal{P} = (\Pi, \Delta)$  and a literal  $\alpha$ , we say that  $\langle \mathcal{A}, \alpha \rangle$  is an argument for  $\alpha$  if  $\mathcal{A}$  is a set of defeasible rules of  $\Delta$  such that:

- (i) there is a derivation for  $\alpha$  from  $\Pi \cup \mathcal{A}$ ,
- (ii) the set  $\Pi \cup \mathcal{A}$  is not contradictory, and
- (iii)  $\mathcal{A}$  is minimal (i.e., there is no proper subset  $\mathcal{A}'$  of  $\mathcal{A}$  satisfying both (i) and (ii)).

## Example

Given  $\mathcal{P}_1$ , we have the following arguments (among others):

- $\langle \mathcal{A}_1, \sim a \rangle = \langle \{(\sim a \prec y), (y \prec x), (x \prec z)\}, \sim a \rangle$
- $\langle \mathcal{A}_2, \sim a \rangle = \langle \{(\sim a \prec y), (y \prec p)\}, \sim a \rangle$
- $\langle \mathcal{A}_3, a \rangle = \langle \{(a \prec w, y), (y \prec p)\}, a \rangle$
- $\langle \mathcal{A}_4, \sim x \rangle = \langle \{(\sim x \prec t, p)\}, \sim x \rangle$
- $\langle \mathcal{A}_5, x \rangle = \langle \{(x \prec p)\}, x \rangle$

# Argument

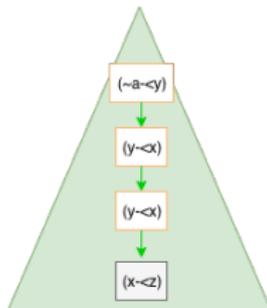
Given a DeLP program  $\mathcal{P} = (\Pi, \Delta)$  and a literal  $\alpha$ , we say that  $\langle \mathcal{A}, \alpha \rangle$  is an argument for  $\alpha$  if  $\mathcal{A}$  is a set of defeasible rules of  $\Delta$  such that:

- (i) there is a derivation for  $\alpha$  from  $\Pi \cup \mathcal{A}$ ,
- (ii) the set  $\Pi \cup \mathcal{A}$  is not contradictory, and
- (iii)  $\mathcal{A}$  is minimal (i.e., there is no proper subset  $\mathcal{A}'$  of  $\mathcal{A}$  satisfying both (i) and (ii)).

**Example (An argument for  $\sim a$  built from the previous program)**

Given  $\mathcal{P}_1$ , we have the following arguments (among others):

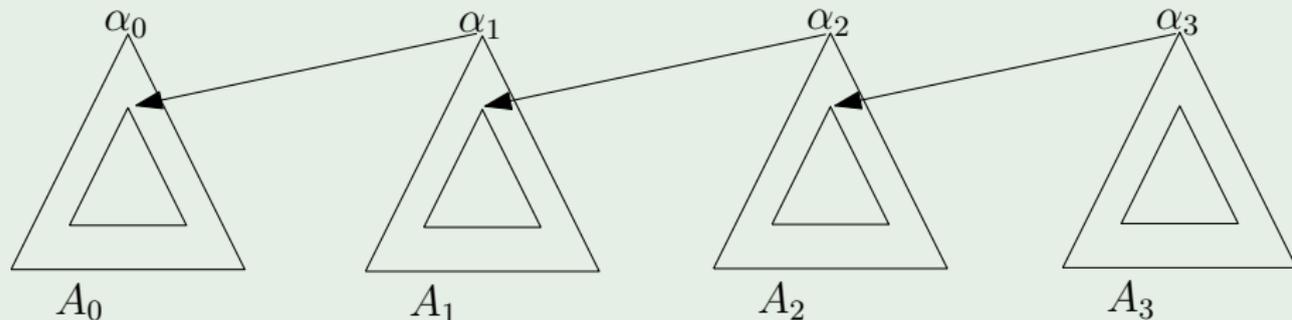
- $\langle \mathcal{A}_1, \sim a \rangle = \langle \{(\sim a \prec y), (y \prec x), (x \prec z)\}, \sim a \rangle$



# Argumentation Line

- A sequence of arguments obtained from a DeLP program, where each element of the sequence is a **defeater** of its predecessor.

## Example (Argumentation Line)



## Example (An argumentation line from the previous program)

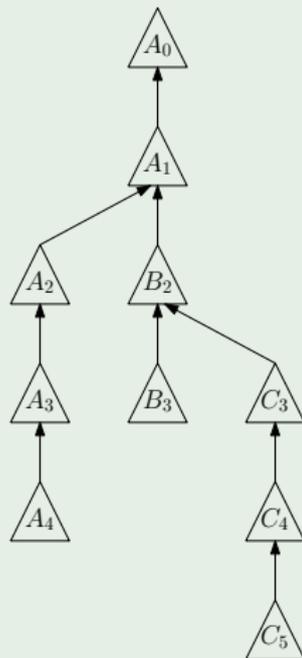
Given  $\mathcal{P}$ , an argumentation line is the following:

$$[A_1, A_3]$$

# Dialectical Process

- Given an argument  $A$  for a literal  $L$ , the dialectical tree contains all acceptable argumentation lines that start with that argument.
- It allows to determine the status for a given argument.

## Example (Dialectical Tree)





# Status of literals

The status of literals allow us to determine the conclusions we can draw from a DeLP-program.

$S_{\mathcal{P}} : Lit \rightarrow \{IN, OUT, UNDECIDED, UNKNOWN\}$  assigning a *status* to each literal w.r.t.  $\mathcal{P}$  as follows:

- $S_{\mathcal{P}}(\alpha) = IN$  if there exists a (marked) dialectical tree whose root  $\alpha$  is *Undefeated*
- $S_{\mathcal{P}}(\alpha) = OUT$  if  $S_{\mathcal{P}}(\sim\alpha) = IN$
- $S_{\mathcal{P}}(\alpha) = UNDECIDED$  if neither  $S_{\mathcal{P}}(\alpha) = IN$  nor  $S_{\mathcal{P}}(\alpha) = OUT$
- $S_{\mathcal{P}}(\alpha) = UNKNOWN$  if  $\alpha \notin Lit_{\mathcal{P}}$ , i.e.,  $\alpha$  is not in the language of the program

## Example (Arguments from the previous program)

Given  $\mathcal{P}$ , then  $S_{\mathcal{P}}(t) = IN$ ,  $S_{\mathcal{P}}(\sim w) = OUT$ ,  $S_{\mathcal{P}}(a) = UNDECIDED$ .

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# Adding a rule to a DeLP program

An update consists of modifying a DeLP-program  $\mathcal{P}$  into a new DeLP-program  $\mathcal{P}'$  by adding a strict or a defeasible rule. In particular,  $\mathcal{P}'$  is as follows:

- If  $r$  is a **strict** rule, then if  $(\Pi \cup r)$  is not contradictory, then  $\mathcal{P}' = ((\Pi \cup r), \Delta)$  otherwise  $\mathcal{P}' = \mathcal{P}$  (i.e., the update has no effect if it would yield a contradictory program).
- If  $r$  is a **defeasible** rule, then  $\mathcal{P}' = (\Pi, (\Delta \cup r))$ , that is, adding a defeasible rule is always permitted.

## Example (Perform $r = (a \prec x)$ on $\mathcal{P}_1$ )

The updated DeLP-program  $\mathcal{P}'_1 = (\Pi'_1, \Delta'_1)$ , is as follows:

$$\Pi'_1 = \Pi_1 = \{w, t, z, (p \leftarrow t)\}$$

$$\Delta'_1 = \Delta_1 = \left\{ \begin{array}{cccc} (\sim a \prec y), & (y \prec x), & (x \prec z), & (\sim x \prec p, t) \\ (a \prec w, y), & (\sim w \prec t), & (y \prec p), & (x \prec p) \end{array} \right\} \cup \{(a \prec x)\}$$

# Question

After performing an update the conclusion that can be derived may change: the status of  $a$  is IN (was UNDECIDED) after performing the update  $r = (y \leftarrow t)$  in our example.

**Should we recompute the status of literals from scratch?**

# Overview of our incremental approach

## Two main steps.

- 1) First, we check if the update is *irrelevant* (the status of all literals are preserved).

In such a case we simply return the initial status  $S_{\mathcal{P}}$ .

- 2) To efficiently deal with *relevant* updates, we identify the subset of literals whose status needs to be recomputed after performing an update, and only recompute their status.

## Irrelevant defeasible-rule update

Adding a rule whose head consists of a literal that already appears as a fact does not affect the status of any other literal of the program.

**Proposition** Let  $\mathcal{P}$  be a DeLP-program, and  $r = \alpha_0 \leftarrow \alpha_1, \dots, \alpha_n$  an update for  $\mathcal{P}$ . If  $S_{\mathcal{P}}(\alpha_0) = \text{IN}$  then  $r$  is irrelevant for  $\mathcal{P}$  (i.e.,  $S_{\mathcal{P}'} = S_{\mathcal{P}}$ ).

However, many updates are *relevant*.

### Example (Relevant update)

Consider the DeLP-program  $\mathcal{P}_1$  from our running example, where we have that  $S_{\mathcal{P}}(y) = S_{\mathcal{P}}(t) = \text{IN}$ . If the rule addition update is  $r = (y \leftarrow t)$ , then  $S_{\mathcal{P}'}(a)$  becomes IN, though it was UNDECIDED before performing the update.

# Hyper-graph for a DeLP-Program

Given a program  $\mathcal{P}$ ,  $G(\mathcal{P}) = \langle N, H \rangle$  is defined as follows:

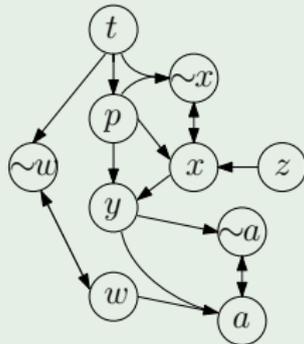
- If there is a strict derivation in  $\Pi$  for literal  $\alpha$ , then  $\alpha \in N$ ;
- For each strict rule  $\alpha_0 \leftarrow \alpha_1, \dots, \alpha_n$  (resp., defeasible rule  $\alpha_0 \prec \alpha_1, \dots, \alpha_n$ ) such that  $\alpha_1, \dots, \alpha_n \in N$ , then  $\alpha_0 \in N$  and  $(\{\alpha_1, \dots, \alpha_n\}, \alpha_0) \in H$ ;
- For each pair of nodes in  $N$  representing complementary literals  $\alpha$  and  $\sim\alpha$ , both  $(\{\alpha\}, \sim\alpha) \in H$  and  $(\{\sim\alpha\}, \alpha) \in H$ .

## Example (Hyper-graph $G(\mathcal{P})$ for $\mathcal{P}$ )

Consider the DeLP-program  $\mathcal{P}_1 = (\Pi_1, \Delta_1)$ , where:

$$\Pi_1 = \{w, t, z, (p \leftarrow t)\}$$

$$\Delta_1 = \left\{ \begin{array}{llll} (\sim a \prec y), & (y \prec x), & (x \prec z), & (\sim x \prec p, t) \\ (a \prec w, y), & (\sim w \prec t), & (y \prec p), & (x \prec p) \end{array} \right\}$$

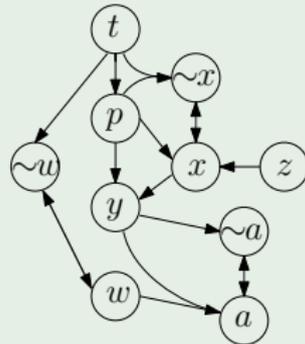


# Preserved Literals

We say that a node  $y$  is *reachable* from a set  $X$  of nodes if there exists a hyper-path from  $X$  to  $y$ . We use  $Reach_{G(\mathcal{P})}(X)$  to denote the set of all nodes that are reachable from  $X$  in  $G(\mathcal{P})$ .

## Example

$$Reach_{G(\mathcal{P})}(\{y\}) = \{y, a, \sim a\}$$



## Theorem (Preserved literals)

Given a DeLP program  $\mathcal{P}$  and an update  $r$ , a literal  $\alpha$  is preserved (i.e.,  $S_{\mathcal{P}}(\alpha) = S_{\mathcal{P}'}(\alpha)$ ) if  $\alpha \notin Reach_{G(\mathcal{P})}(\{head(r)\})$ .

# Incremental Algorithm

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## Algorithm Dynamic DeLP-Solver

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**Input:** DeLP-program  $\mathcal{P}$ , Initial status  $S_{\mathcal{P}}$ , Update  $r$ .

**Output:** Status  $S_{\mathcal{P}'}$  of the updated program  $\mathcal{P}'$ .

- 1: **if** the update is irrelevant **then**
  - 2:   **return**  $S_{\mathcal{P}}$ ;
  - 3: **if**  $head(r)$  is a fresh literal **then**
  - 4:   **return**  $S_{\mathcal{P}} \cup \left\{ \left( head(r), DELP-SOLVER(\mathcal{P}', head(r)) \right) \right\}$
  - 5: Let  $G(\mathcal{P}) = \langle N, H \rangle$ ; // Build Hyper-graph
  - 6: Let  $\mathcal{R} = Reach_{G(\mathcal{P})}(\{head(r)\})$ ; // Literals not preserved
  - 7: Let  $\overline{\mathcal{R}} = \{\alpha \in (N \setminus \mathcal{R})\}$ ; // Preserved literals
  - 8: **for**  $\alpha$  in  $\overline{\mathcal{R}}$  **do**
  - 9:    $S_{\mathcal{P}'}(\alpha) \leftarrow S_{\mathcal{P}}(\alpha)$  // Copy the status of preserved literals
  - 10: **for**  $\alpha$  in  $\mathcal{R}$  **do**
  - 11:    $S_{\mathcal{P}'}(\alpha) \leftarrow DELP-SOLVER(\mathcal{P}', \alpha)$  // Recompute the status of literals
  - 12: **return**  $S_{\mathcal{P}'}$ .
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# Dataset & Metodology

## Datasets:

We randomly generated a dataset of 30 DeLP-programs, each of them having 200 positive and 20 negative literals, 8 facts, a number of strict rules in  $\{5, 15, \dots, 35\}$ , and a number  $d$  of defeasible rules in  $\{100, 120, \dots, 200\}$ , where each (strict or defeasible) rule has a number of literals in the body in  $\{1, 2, 3\}$ .

## Methodology

For each DeLP-program  $\mathcal{P}$  in the dataset, we compared the average running time of Algorithm 1 with that of the approach from scratch, which computes the status in the updated program by directly calling the DeLP-Solver for each literal of  $\mathcal{P}$ .

# Dataset & Methodology

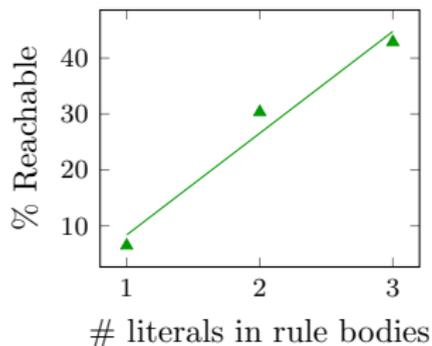
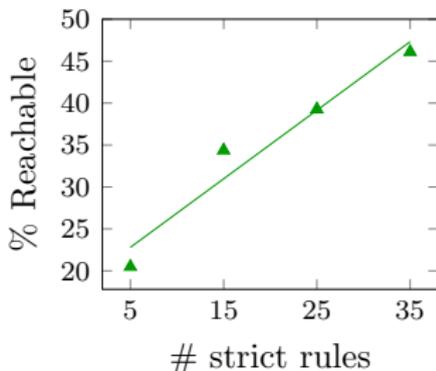
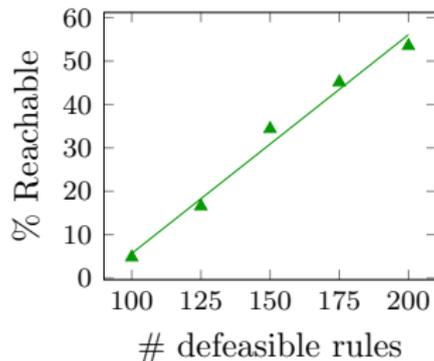
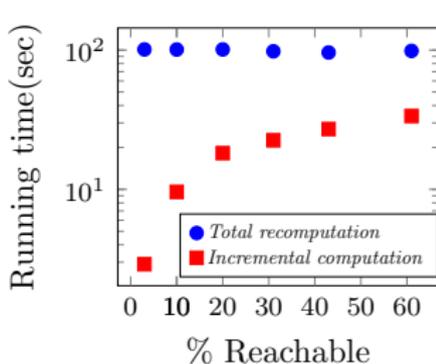
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# Experimental Results



# Results

- 1) We compared our technique with the computation from scratch.
- 2) Our incremental algorithm outperforms the computation from scratch.
- 3) Our technique is sensitive to the percentage of literals reachable from the update: the higher it is the lower the benefits are.
- 4) A study to determine which parameter impacts on reachability is reported.

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- \* We have taken the first steps in tackling the problem of avoiding wasted effort when determining the warrant status of literals in a DeLP program after that a (defeasible or strict) rule is added
- \* Our incremental approach outperforms the computation from scratch (especially if the average number of literals reachable from an update is less than 30%).

FW1) Further developing these techniques, as well as developing similar ones for rule deletion, fact addition and deletion, and the more general case of simultaneously adding or deleting a *set* of rules and facts.

FW2) Investigating how our technique can be also extended to cope with other structured argumentation frameworks.

Thank you!

... any ~~question~~ **argument**?