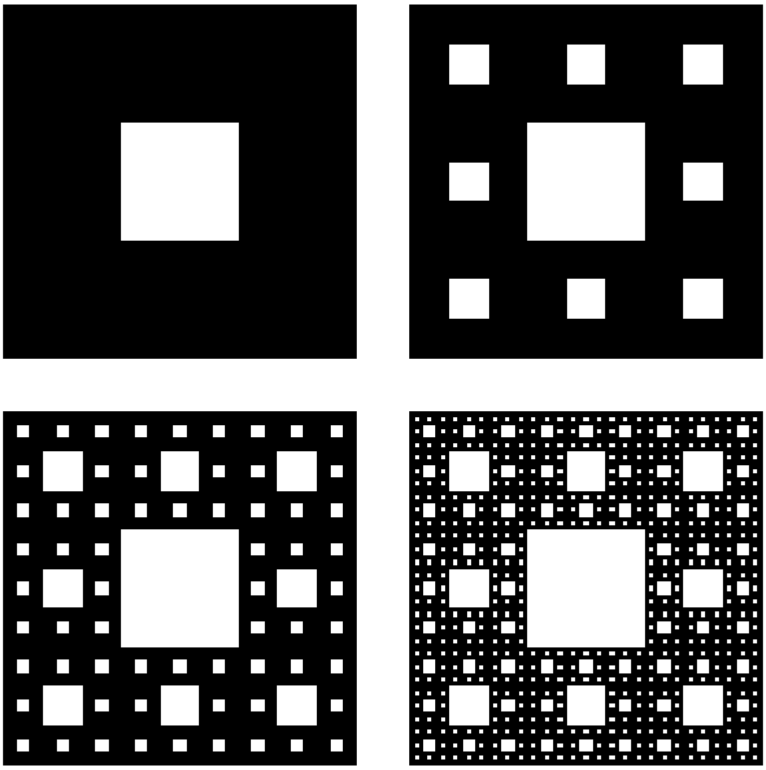
Published by MIT

03/19/2012

'Infinity Computer' Calculates Area Of Sierpinski Carpet Exactly

Mathematicians have never been comfortable handling infinities, such as those that crop up in the area of a Sierpinski carpet. But an entirely new type of mathematics looks set to by-pass the problem

By kfc



A Sierpinksi carpet is one of the more famous fractal objects in mathematics. Creating one is an iterative procedure. Start with a square, divide it into nine equal squares and remove the central one. That leaves eight squares around a central square hole.

In the next iteration, repeat this process with each of the eight remaining squares and so on (see above).

One interesting problem is to find the area of a Sierpinski triangle. Clearly this changes with each iteration. Assuming the original square has area equal to 1, the area after the first iteration is 8/9. After the second iteration, it is (8/9)^2; after the third it is (8/9)^3 and so on.

So the area of a Sierpinski carpet after n iterations is (8/9)^n. That's straightforward.

But what is the area of the carpet after an infinite number of iterations?

Ordinary mathematics has no answer to this question because it lacks the tools for handling infinity. Instead, mathematicians look at the properties of the mathematical system and how it behaves as it tends towards infinity. They even have plenty of formal tools for exploring these limits. But the properties at infinity have to be assumed.

In this case, the area of the carpet tends to zero as the number of iterations tends to infinity so the area of a Sierpinski carpet is zero.

That leaves many mathematicians with a sour taste in their mouths. The reason is that the area of a Sierpinski carpet close to infinity ought to be highly sensitive to its original shape, whether a square or some other pattern. But the process of finding the limits blurs this behaviour.

For example, instead of starting with a square, imagine starting with the shape in the top left had corner of the figure above, let's call it a square doughnut. The square donut consists of eight squares, each with sides of length 1/3. Obviously, the area of this Sierpinksi carpet tends to zero as n tends to infinity.

But the square doughnut carpet is one step ahead of the traditional Sierpinski carpet but that gets lost in the traditional approach. At infinity they are treated as equal.

If that doesn't sound very significant, imagine running the process in reverse, starting from infinity and working backwards to end up with a square or a square doughnut or some other shape in the carpet sequence.

In that case, each shape can be created by the same (infinite) number of steps so it's not possible to distinguish between them. That's clearly absurd.

Today, Yaroslav Sergeyev, a mathematician at the University of Calabria in Italy solves this problem (and the analogous three dimensional version called Menger's sponge).

For the last few years, Sergeyev has been championing a new type of mathematics called infinity computing. The basic idea is to replace the notion of infinity with a new number that Sergeyev calls grossone, which he writes like this:

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Sergeyev begins by adding a new axiom to the axiom of real numbers, which he calls the infinite unit axiom. This introduces grossone - the infinite unit.

Because it is governed by the other axioms of real numbers, grossone behaves much like one too. So it's possible to multiply grossone, divide it, add to it and subtract from it, just as is possible with other real numbers.

That suddenly makes working at infinity much easier by using a computing process that Sergeyev calls the infinity computer, which has the additional axiom built in. "The introduction of grossone gives a possibility to work with finite, infinite and infinitesimal quantities numerically," he says.

To show off its power, he works through the Sierpinski carpet examples given above, revealing how it's possible to keep track of the number of iterations at infinity simply by adding or subtracting real numbers from grossone. If a square can created in grossone steps, a square doughnut can be created in -grossone minus 1- steps. In this way, it's a simple matter to differentiate between any of the shapes in carpet sequence.

That looks handy. The inability to keep track of mathematical processes at or close to infinity in a consistent fashion has frustrated mathematicians and physicists for centuries.

So if Sergeyev has found a way round this that works, that's clearly a highly significant advance.

Ref: arxiv.org/abs/1203.3150: Evaluating The Exact Infinitesimal Values Of Area Of Sierpinski's Carpet And Volume Of Menger's Sponge