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Dimensional Inconsistency Measures and Postulates in Spatio-Temporal Databases (Extended Abstract)

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Introduction

- Spatio-temporal (ST) databases

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- Inconsistency measures (IMs) for ST databases
  - Dimensional IMs
  - Repair-based IMs

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- Postulates
  - Classical
  - Dimensional

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- Inconsistency measures (IMs) for ST databases
  - Dimensional IMs
  - Repair-based IMs
- Postulates
  - Classical
  - Dimensional
- Postulate satisfaction
- Complexity analysis

ST databases

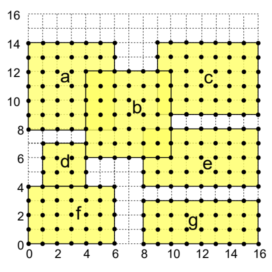
Declarative framework for the representation and processing of spatio-temporal data

ST databases

Declarative framework for the representation and processing of spatio-temporal data

Example

Id	Region	Time
id_1	*a*	1
id_1	*b*	1
id_1	*g*	3
id_2	*b*	2
id_2	*e*	2
id_3	*c*	1



ST databases

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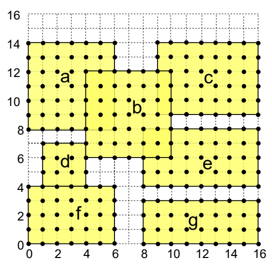

Assumptions:

- An object can be in only one location at a time
- A location may contain more than one object

ST databases - **Syntax**

Notation

- *ID* is a set of object ids
- *Space* is a set of point locations
- *T* is a set of integer time values

Definition

An **ST atom** is a tuple (id, r, t) where

- $id \in ID$ is **an object id**,
- $\emptyset \subsetneq r \subseteq Space$ is a **region**,
- $t \in T$ is a **time point.**

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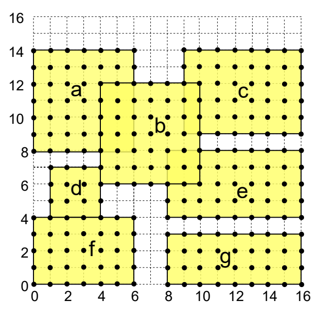
Definition

An **ST database** is a finite set of ST atoms.

ST databases - **Syntax**

Example



An ST database:

$(id_1, a, 1)$
$(id_1, b, 1)$
$(id_3, c, 1)$
$(id_2, b, 2)$
$(id_2, e, 2)$
$(id_1, g, 3)$

ST databases - **Semantics**

Definition

An **ST interpretation** I is a function $I : ID \times T \rightarrow Space$.

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An ST interpretation I is a **model** for an ST database $\mathcal{S}$ iff I satisfies every ST atom in $\mathcal{S}$.

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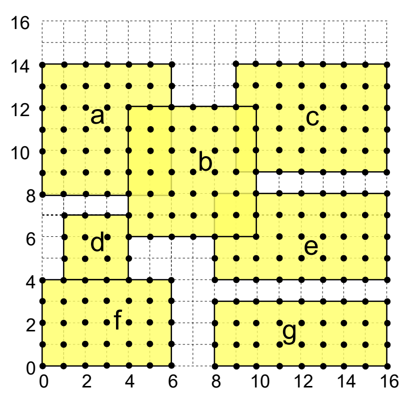
An ST interpretation I is a **model** for an ST database $\mathcal{S}$ iff I satisfies every ST atom in $\mathcal{S}$.

Definition

An ST database $\mathcal{S}$ is **consistent** iff there exists at least one model for $\mathcal{S}$, otherwise $\mathcal{S}$ is **inconsistent**.

ST databases - **Semantics**

Example



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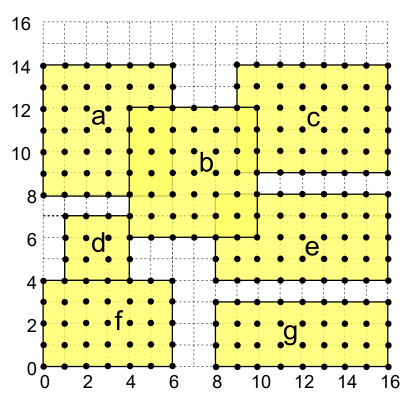

ST database $\mathcal{S}$:	ST interpretation I:
$(id_1, a, 1)$	$I(id_1, 1) = (5, 10)$
$(id_1, b, 1)$	$I(id_2, 1) = (1, 9)$
$(id_3, c, 1)$	$I(id_3, 1) = (10, 10)$
$(id_2, b, 2)$	$I(id_1, 2) = (9, 10)$
$(id_2, e, 2)$	$I(id_2, 2) = (9, 7)$
$(id_1, g, 3)$	$I(id_3, 2) = (12, 10)$
	$I(id_1, 3) = (10, 2)$
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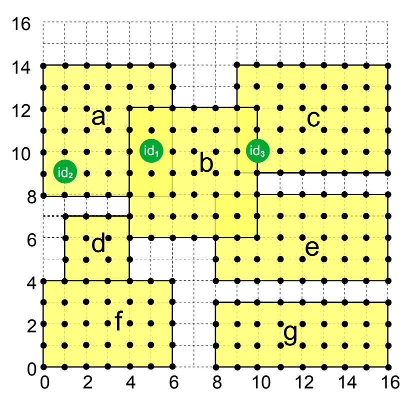
	$I(id_3, 3) = (12, 7)$

ST databases - **Semantics**

Example



Time point 1

ST database $\mathcal{S}$:

$(\mathbf{id_1}, \mathbf{a}, \mathbf{1})$
$(\mathbf{id_1}, \mathbf{b}, \mathbf{1})$
$(\mathbf{id_3}, \mathbf{c}, \mathbf{1})$
$(id_2, b, 2)$
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$(id_1, g, 3)$

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ST databases - **Semantics**

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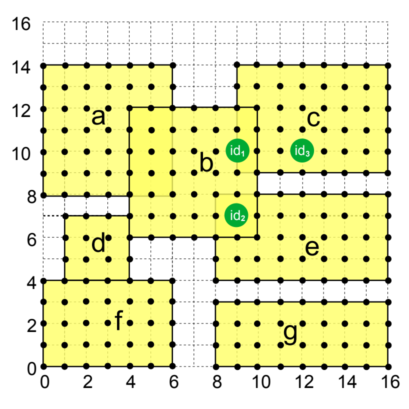

Time point 2

ST database $\mathcal{S}$:

$(id_1, a, 1)$
$(id_1, b, 1)$
$(id_3, c, 1)$
$\mathbf{(id_2, b, 2)}$
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ST databases - **Semantics**

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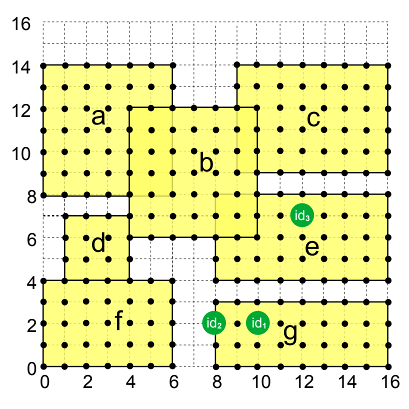

Time point 3

ST database $\mathcal{S}$:

$(id_1, a, 1)$
$(id_1, b, 1)$
$(id_3, c, 1)$
$(id_2, b, 2)$
$(id_2, e, 2)$
$(\mathbf{id_1}, \mathbf{g}, \mathbf{3})$

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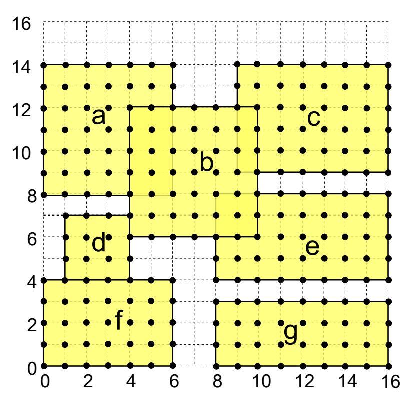

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I is a **model** for $\mathcal{S} \Rightarrow \mathcal{S}$ is **consistent**

ST databases - **Semantics**

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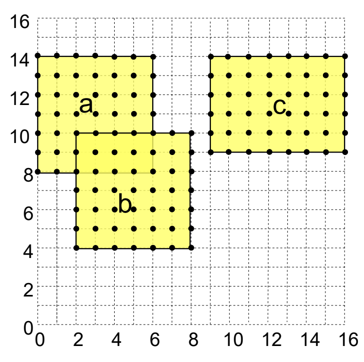

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ST databases - **Semantics**

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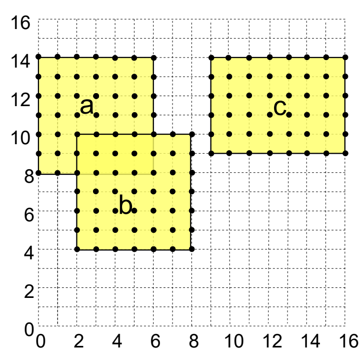

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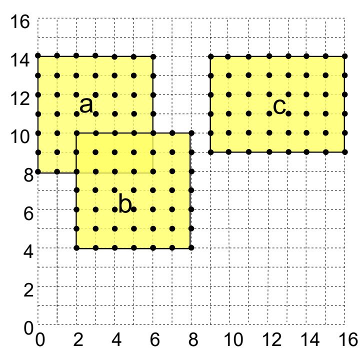

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There is **NO model** for $\mathcal{S} \Rightarrow \mathcal{S}$ is **inconsistent**

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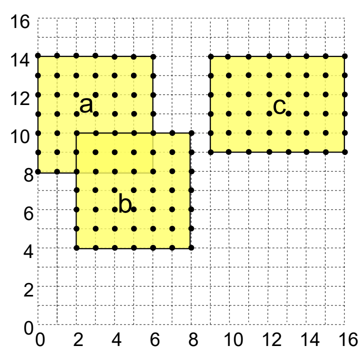

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Minimal Inconsistent Subset (MIS):
Inclusion-minimal inconsistent subset of the ST database

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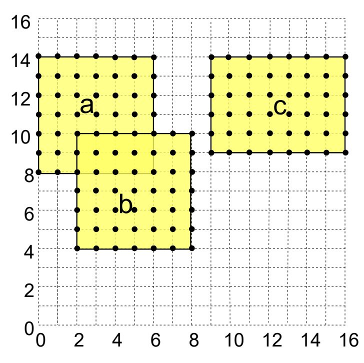

ST database $\mathcal{S}$:

$\mathbf{(id_1, b, 1)}$
$\mathbf{(id_1, c, 1)}$ } MIS_1

$\mathbf{(id_1, a, 2)}$
$\mathbf{(id_1, c, 2)}$ } MIS_2

$(id_2, a, 1)$
$(id_2, b, 1)$

There is **NO model** for $\mathcal{S} \Rightarrow \mathcal{S}$ is **inconsistent**

Minimal Inconsistent Subset (MIS):
Inclusion-minimal inconsistent subset of the ST database

Inconsistency Measures (IMs)

Definition

Inconsistency measure. A function $\mathcal{I}$:

Input: An ST database $\mathcal{S}$
Output: A non-negative real number $\mathcal{I}(\mathcal{S})$

such that the following two properties hold:

1. **(Consistency)** $\mathcal{I}(\mathcal{S}) = 0$ iff $\mathcal{S}$ is consistent.
2. **(Monotony)** if $\mathcal{S} \subseteq \mathcal{S}'$, then $\mathcal{I}(\mathcal{S}) \leq \mathcal{I}(\mathcal{S}')$.

Postulates

- $\mathcal{M}(\mathcal{S})$: Set of all MISs of $\mathcal{S}$
- $\mathit{Problematic}(\mathcal{S}) = \bigcup_{M \in \mathcal{M}(\mathcal{S})} M$
- $\mathit{Free}(\mathcal{S}) = \mathcal{S} \setminus \mathit{Problematic}(\mathcal{S})$

Definition

Let $\mathcal{I}$ be an IM, and $\mathcal{S}$, $\mathcal{S}'$ be two ST databases.

1. **(Free-Formula Independence)** If $(id, r, t) \in \mathit{Free}(\mathcal{S})$ then $\mathcal{I}(\mathcal{S}) = \mathcal{I}(\mathcal{S} \setminus \{(id, r, t)\})$.
2. **(Penalty)** If $(id, r, t) \in \mathit{Problematic}(\mathcal{S})$ then $\mathcal{I}(\mathcal{S}) > \mathcal{I}(\mathcal{S} \setminus \{(id, r, t)\})$.
3. **(Dominance)** If (id, r, t) and (id, r', t) are ST atoms such that $r \subseteq r'$ then $\mathcal{I}(\mathcal{S} \cup \{(id, r, t)\}) \geq \mathcal{I}(\mathcal{S} \cup \{(id, r', t)\})$.
4. **(Super-Additivity)** If $\mathcal{S} \cap \mathcal{S}' = \emptyset$ then $\mathcal{I}(\mathcal{S} \cup \mathcal{S}') \geq \mathcal{I}(\mathcal{S}) + \mathcal{I}(\mathcal{S}')$.
5. **(Attenuation)** If $M, M' \in \mathcal{M}(\mathcal{S})$ and $|M| < |M'|$ then $\mathcal{I}(M) > \mathcal{I}(M')$.
6. **(Equal Conflict)** If $M, M' \in \mathcal{M}(\mathcal{S})$ and $|M| = |M'|$ then $\mathcal{I}(M) = \mathcal{I}(M')$.
7. **(MI-Normalization)** If $M \in \mathcal{M}(\mathcal{S})$ then $\mathcal{I}(M) = 1$.
8. **(MI-Separability)** If $\mathcal{M}(\mathcal{S} \cup \mathcal{S}') = \mathcal{M}(\mathcal{S}) \cup \mathcal{M}(\mathcal{S}')$ and $\mathcal{M}(\mathcal{S}) \cap \mathcal{M}(\mathcal{S}') = \emptyset$ then $\mathcal{I}(\mathcal{S} \cup \mathcal{S}') = \mathcal{I}(\mathcal{S}) + \mathcal{I}(\mathcal{S}')$.

Dimensional IMs

- Measuring inconsistency along the **object** dimension
- Measuring inconsistency along the **time** dimension
- Measuring inconsistency along the **object** and **time** dimensions **together**
- Measuring inconsistency along the **space** dimension (2 IMs)

Dimensional IMs – **Object** dimension

Idea: Count how many **objects** are involved in an inconsistency

$$\mathcal{I}_O(\mathcal{S}) = |\{id \in ID \mid (id, r, t) \text{ belongs to a MIS of } \mathcal{S}\}|$$

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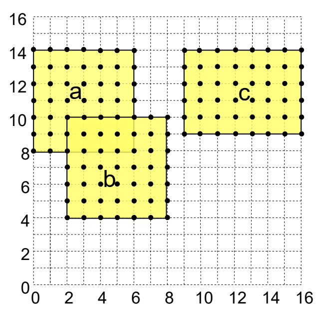

ST database $\mathcal{S}$:

$$\left.\begin{array}{l}(\mathbf{id_1}, b, 1)\\(\mathbf{id_1}, c, 1)\end{array}\right\} MIS_1$$

$$\left.\begin{array}{l}(\mathbf{id_1}, a, 2)\\(\mathbf{id_1}, c, 2)\end{array}\right\} MIS_2$$

$(id_2, a, 1)$

$(id_2, b, 1)$

$$\mathcal{I}_O(\mathcal{S}) = |\{id_1\}| = \mathbf{1}$$

Dimensional IMs – **Time** dimension

Idea: Count how many **time values** are involved in an inconsistency

$$\mathcal{I}_T(\mathcal{S}) = |\{t \in T \mid (id, r, t) \text{ belongs to a MIS of } \mathcal{S}\}|$$

Dimensional IMs – **Time** dimension

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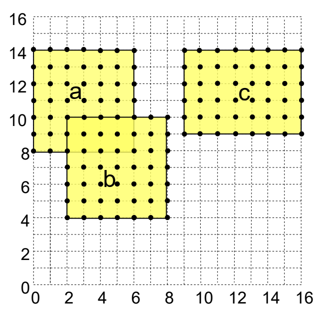

ST database $\mathcal{S}$:

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$(id_1, c, \mathbf{1})$ } MIS_1
$(id_1, a, \mathbf{2})$
$(id_1, c, \mathbf{2})$ } MIS_2
$(id_2, a, 1)$
$(id_2, b, 1)$

$$\mathcal{I}_T(\mathcal{S}) = |\{1, 2\}| = \mathbf{2}$$

Dimensional IMs – **Object + Time** dimensions

Idea: Count how many **(id, time-value) pairs** are involved in an inconsistency

$$\mathcal{I}_{OT}(\mathcal{S}) = |\{(id, t) \mid (id, r, t) \text{ belongs to a MIS of } \mathcal{S}\}|$$

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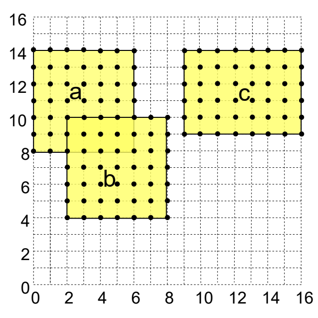

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$(\mathbf{id_1}, a, \mathbf{2})$
$(\mathbf{id_1}, c, \mathbf{2})$ } MIS_2
$(id_2, a, 1)$
$(id_2, b, 1)$

$$\mathcal{I}_{OT}(\mathcal{S}) = |\{(id_1, 1), (id_1, 2)\}| = \mathbf{2}$$

Dimensional IMs – **Space** dimension (1)

Idea: Count how many **points are in regions** that are involved in an inconsistency

$$\mathcal{I}_S(\mathcal{S}) = |\bigcup \{r \mid (id, r, t) \text{ belongs to a MIS of } \mathcal{S}\}|$$

Dimensional IMs – **Space** dimension (1)

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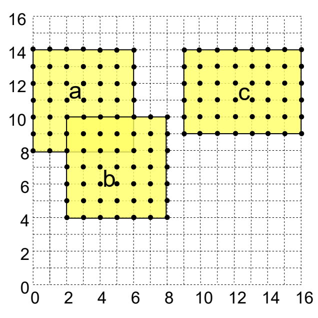

ST database $\mathcal{S}$:

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$(id_1, \mathbf{a}, 2)$
$(id_1, \mathbf{c}, 2)$ } MIS_2
$(id_2, a, 1)$
$(id_2, b, 1)$

$$\mathcal{I}_S(\mathcal{S}) = |a \cup b \cup c| = \mathbf{131}$$

Dimensional IMs – **Space** dimension (2)

Idea: Sum up the **"distances"** of all MISs.

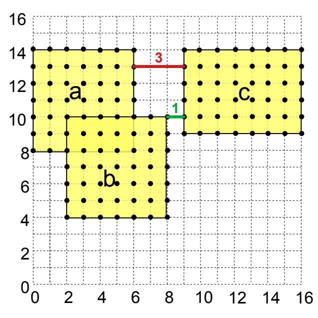
$$\mathcal{I}_D(\mathcal{S}) = \sum_{M \in \mathcal{M}(\mathcal{S})} d(M)$$

Dimensional IMs – **Space** dimension (2)

Idea: Sum up the "**distances**" of all MISs.

$$\mathcal{I}_D(\mathcal{S}) = \sum_{M \in \mathcal{M}(\mathcal{S})} d(M)$$

Example



ST database $\mathcal{S}$:

$(id_1, \mathbf{b}, 1)$
$(id_1, \mathbf{c}, 1)$ } MIS_1, $\mathbf{d}(MIS_\mathbf{1}) = \mathbf{1}$

$(id_1, \mathbf{a}, 2)$
$(id_1, \mathbf{c}, 2)$ } MIS_2, $\mathbf{d}(MIS_\mathbf{2}) = \mathbf{3}$

$(id_2, a, 1)$
$(id_2, b, 1)$

$$\mathcal{I}_D(\mathcal{S}) = \mathbf{1} + \mathbf{3} = \mathbf{4}$$

Repair-based IMs

Main idea: Measure the cost of restoring consistency along the **object**, **time**, and **space** dimensions

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Additionally, measure the cost of restoring consistency by deleting **entire ST atoms** (so, not dimensional)

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Additionally, measure the cost of restoring consistency by deleting **entire ST atoms** (so, not dimensional)

How to restore consistency?

- By changing only **object ids**
- By changing only **time values**
- By changing only **regions**
- By deleting entire ST atoms

Repair-based IMs

Main idea: Measure the cost of restoring consistency along the **object**, **time**, and **space** dimensions

Additionally, measure the cost of restoring consistency by deleting **entire ST atoms** (so, not dimensional)

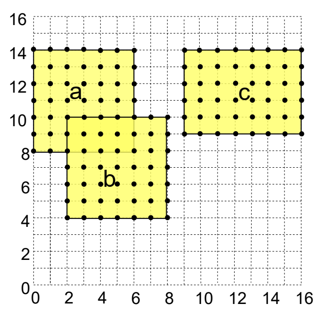
How to restore consistency?

- By changing only **object ids**
- By changing only **time values**
- By changing only **regions**
- By deleting entire ST atoms

Each change has a cost
IM value: minimum overall cost of changes needed to restore consistency

Repair-based IMs – **Object** dimension

Example



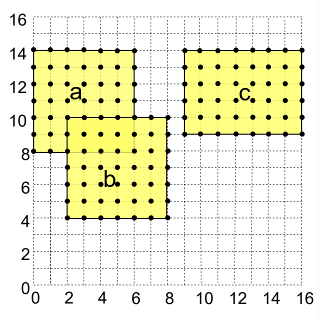
(inconsistent)

ST database $\mathcal{S}$:

$(id_1, b, 1)$
$(id_1, c, 1)$
$(id_1, a, 2)$
$(id_1, c, 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

Repair-based IMs – **Object** dimension

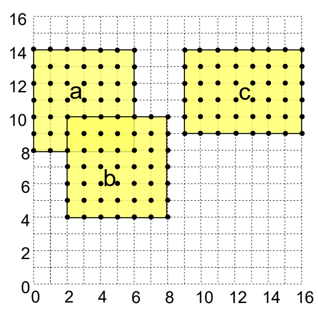
Example



(inconsistent)	**(consistent)**
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:
$(\mathbf{id_1}, b, 1)$	$(\mathbf{id_2}, b, 1)$
$(id_1, c, 1)$	$(id_1, c, 1)$
$(\mathbf{id_1}, a, 2)$	$(\mathbf{id_2}, a, 2)$
$(id_1, c, 2)$	$(id_1, c, 2)$
$(id_2, a, 1)$	$(id_2, a, 1)$
$(id_2, b, 1)$	$(id_2, b, 1)$

Repair-based IMs – **Object** dimension

Example

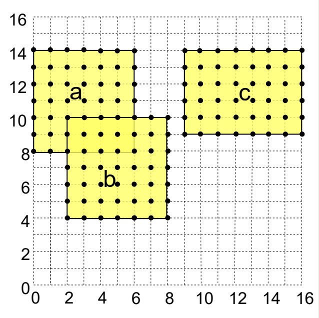


(inconsistent) ST database $\mathcal{S}$:	**(consistent)** ST database $\mathcal{S}'$:	
$(\mathbf{id_1}, b, 1)$	$(\mathbf{id_2}, b, 1)$	**cost 1**
$(id_1, c, 1)$	$(id_1, c, 1)$	
$(\mathbf{id_1}, a, 2)$	$(\mathbf{id_2}, a, 2)$	**cost 1**
$(id_1, c, 2)$	$(id_1, c, 2)$	
$(id_2, a, 1)$	$(id_2, a, 1)$	
$(id_2, b, 1)$	$(id_2, b, 1)$	

Assuming the cost to change *id* into *id′* is their **edit distance**

Repair-based IMs – **Object** dimension

Example



(inconsistent)

ST database $\mathcal{S}$:

$(\mathbf{id_1}, b, 1)$
$(id_1, c, 1)$
$(\mathbf{id_1}, a, 2)$
$(id_1, c, 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

(consistent)

ST database $\mathcal{S}'$:

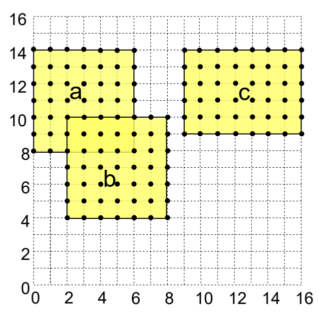
$(\mathbf{id_2}, b, 1)$ **cost 1**
$(id_1, c, 1)$
$(\mathbf{id_2}, a, 2)$ **cost 1**
$(id_1, c, 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

Assuming the cost to change *id* into *id′* is their **edit distance**

$$\mathcal{I}_{id}(\mathcal{S}) = \mathbf{1} + \mathbf{1} = \mathbf{2}$$

Repair-based IMs – **Time** dimension

Example



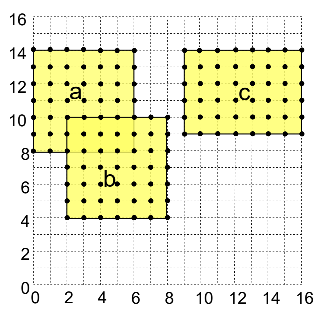
(inconsistent)

ST database $\mathcal{S}$:

$(id_1, b, 1)$
$(id_1, c, 1)$
$(id_1, a, 2)$
$(id_1, c, 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

Repair-based IMs – **Time** dimension

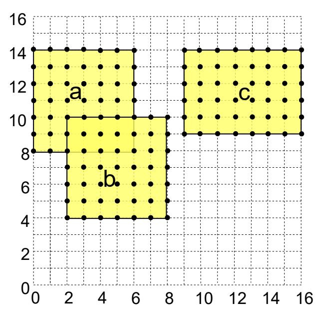
Example



(inconsistent)	**(consistent)**
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:
$(id_1, b, 1)$	$(id_1, b, 1)$
$(id_1, c, \mathbf{1})$	$(id_1, c, \mathbf{2})$
$(id_1, a, \mathbf{2})$	$(id_1, a, \mathbf{1})$
$(id_1, c, 2)$	$(id_1, c, 2)$
$(id_2, a, 1)$	$(id_2, a, 1)$
$(id_2, b, 1)$	$(id_2, b, 1)$

Repair-based IMs – **Time** dimension

Example

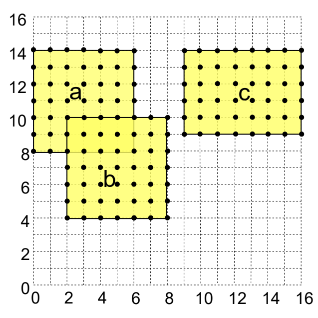


(inconsistent)	**(consistent)**	
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:	
$(id_1, b, 1)$	$(id_1, b, 1)$	
$(id_1, c, \mathbf{1})$	$(id_1, c, \mathbf{2})$	**cost 1**
$(id_1, a, \mathbf{2})$	$(id_1, a, \mathbf{1})$	**cost 1**
$(id_1, c, 2)$	$(id_1, c, 2)$	
$(id_2, a, 1)$	$(id_2, a, 1)$	
$(id_2, b, 1)$	$(id_2, b, 1)$	

Assuming the cost to change t into t' is $|t - t'|$

Repair-based IMs – **Time** dimension

Example



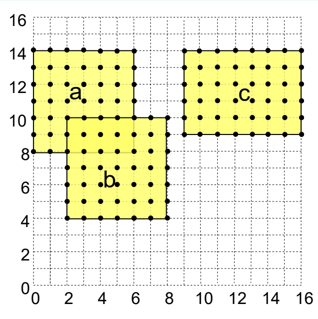
(inconsistent)	**(consistent)**	
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:	
$(id_1, b, 1)$	$(id_1, b, 1)$	
$(id_1, c, \mathbf{1})$	$(id_1, c, \mathbf{2})$	**cost 1**
$(id_1, a, \mathbf{2})$	$(id_1, a, \mathbf{1})$	**cost 1**
$(id_1, c, 2)$	$(id_1, c, 2)$	
$(id_2, a, 1)$	$(id_2, a, 1)$	
$(id_2, b, 1)$	$(id_2, b, 1)$	

Assuming the cost to change t into t' is $|t - t'|$

$$\mathcal{I}_{time}(\mathcal{S}) = \mathbf{1} + \mathbf{1} = \mathbf{2}$$

Repair-based IMs – **Space** dimension

Example



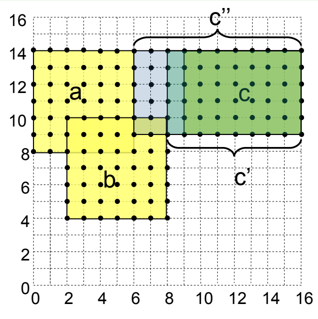
(inconsistent)

ST database $\mathcal{S}$:

$(id_1, b, 1)$
$(id_1, c, 1)$
$(id_1, a, 2)$
$(id_1, c, 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

Repair-based IMs – **Space** dimension

Example



(inconsistent)

ST database $\mathcal{S}$:

$(id_1, b, 1)$
$(id_1, \mathbf{c}, 1)$
$(id_1, a, 2)$
$(id_1, \mathbf{c}, 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

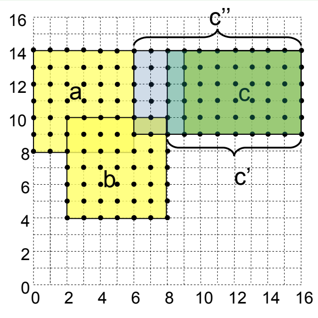
(consistent)

ST database $\mathcal{S}'$:

$(id_1, b, 1)$
$(id_1, \mathbf{c}', 1)$
$(id_1, a, 2)$
$(id_1, \mathbf{c}'', 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

Repair-based IMs – **Space** dimension

Example

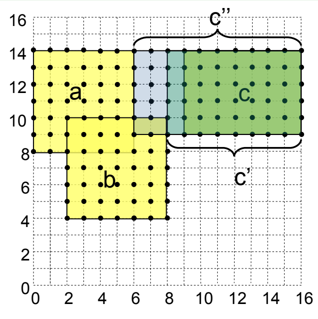


(inconsistent)	**(consistent)**	
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:	
$(id_1, b, 1)$	$(id_1, b, 1)$	
$(id_1, \mathbf{c}, 1)$	$(id_1, \mathbf{c}', 1)$	**cost 6**
$(id_1, a, 2)$	$(id_1, a, 2)$	
$(id_1, \mathbf{c}, 2)$	$(id_1, \mathbf{c}'', 2)$	**cost 18**
$(id_2, a, 1)$	$(id_2, a, 1)$	
$(id_2, b, 1)$	$(id_2, b, 1)$	

Assuming regions must be rectangles and the cost to change r into r' is their **symmetric difference**

Repair-based IMs – **Space** dimension

Example



(inconsistent)	**(consistent)**	
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:	
$(id_1, b, 1)$	$(id_1, b, 1)$	
$(id_1, \mathbf{c}, 1)$	$(id_1, \mathbf{c}', 1)$	**cost 6**
$(id_1, a, 2)$	$(id_1, a, 2)$	
$(id_1, \mathbf{c}, 2)$	$(id_1, \mathbf{c}'', 2)$	**cost 18**
$(id_2, a, 1)$	$(id_2, a, 1)$	
$(id_2, b, 1)$	$(id_2, b, 1)$	

Assuming regions must be rectangles and the cost to change r into r' is their **symmetric difference**

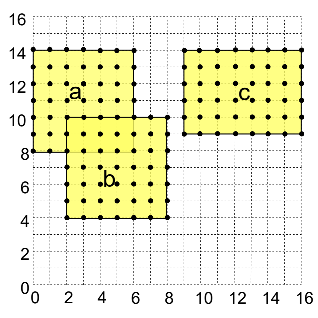
$$\mathcal{I}_{region}(\mathcal{S}) = \mathbf{6} + \mathbf{18} = \mathbf{24}$$

Repair-based IMs – Cardinality criterion

$$\mathcal{I}_{card}(\mathcal{S}) =$$

minimum number of ST atoms to **delete** from $\mathcal{S}$ to restore **consistency**

Example



(inconsistent)

ST database $\mathcal{S}$:

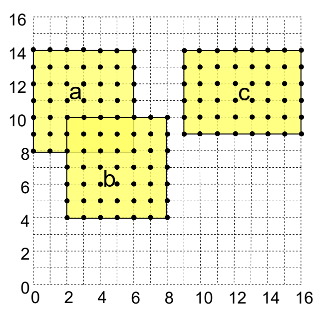
$(id_1, b, 1)$
$(id_1, c, 1)$
$(id_1, a, 2)$
$(id_1, c, 2)$
$(id_2, a, 1)$
$(id_2, b, 1)$

Repair-based IMs – Cardinality criterion

$$\mathcal{I}_{card}(\mathcal{S}) =$$

minimum number of ST atoms to **delete** from $\mathcal{S}$ to restore **consistency**

Example



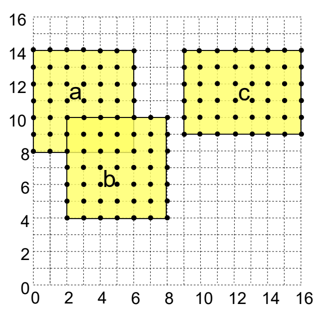
(inconsistent)	**(consistent)**
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:
$(id_1, b, 1)$	~~$(id_1, b, 1)$~~
$(id_1, c, 1)$	$(id_1, c, 1)$
$(id_1, a, 2)$	~~$(id_1, a, 2)$~~
$(id_1, c, 2)$	$(id_1, c, 2)$
$(id_2, a, 1)$	$(id_2, a, 1)$
$(id_2, b, 1)$	$(id_2, b, 1)$

Repair-based IMs – Cardinality criterion

$$\mathcal{I}_{card}(\mathcal{S}) =$$

minimum number of ST atoms to **delete** from $\mathcal{S}$ to restore **consistency**

Example



(inconsistent)	**(consistent)**
ST database $\mathcal{S}$:	ST database $\mathcal{S}'$:
$(id_1, b, 1)$	~~$(id_1, b, 1)$~~
$(id_1, c, 1)$	$(id_1, c, 1)$
$(id_1, a, 2)$	~~$(id_1, a, 2)$~~
$(id_1, c, 2)$	$(id_1, c, 2)$
$(id_2, a, 1)$	$(id_2, a, 1)$
$(id_2, b, 1)$	$(id_2, b, 1)$

$$\mathcal{I}_{card}(\mathcal{S}) = \mathbf{2}$$

Dimensional Postulates

Definition (Dimensional Penalty)

Let $\mathcal{I}$ be an IM and $\mathcal{S}$ be an ST database.

1. **(Object Penalty)** If $(id, r, t) \in \mathit{Problematic}(\mathcal{S})$ and $A = \{(id, r', t') \in \mathit{Problematic}(\mathcal{S})\}$ then $\mathcal{I}(\mathcal{S}) > \mathcal{I}(\mathcal{S} \setminus A)$.
2. **(Time Penalty)** If $(id, r, t) \in \mathit{Problematic}(\mathcal{S})$ and $A = \{(id', r', t) \in \mathit{Problematic}(\mathcal{S})\}$ then $\mathcal{I}(\mathcal{S}) > \mathcal{I}(\mathcal{S} \setminus A)$.
3. **(Space Penalty)** If $(id, r, t) \in \mathit{Problematic}(\mathcal{S})$ and $A = \{(id', r', t') \in \mathit{Problematic}(\mathcal{S}) \mid r \cap r' \neq \emptyset\}$ then $\mathcal{I}(\mathcal{S}) > \mathcal{I}(\mathcal{S} \setminus A)$.

Dimensional Postulates

- $ID(\mathcal{S}) = \{id \mid (id, r, t) \in \mathcal{S}\}$
- $Time(\mathcal{S}) = \{t \mid (id, r, t) \in \mathcal{S}\}$
- $Region(\mathcal{S}) = \bigcup\{r \mid (id, r, t) \in \mathcal{S}\}$

Definition (Dimensional Super-Additivity)

Let $\mathcal{I}$ be an IM, and $\mathcal{S}, \mathcal{S}'$ be two ST databases.
For $X \in \{ID, Time, Region\}$,
if $X(\mathcal{S}) \cap X(\mathcal{S}') = \emptyset$ then $\mathcal{I}(\mathcal{S} \cup \mathcal{S}') \geq \mathcal{I}(\mathcal{S}) + \mathcal{I}(\mathcal{S}')$.

Definition (Dimensional MI-Separability)

Let $\mathcal{I}$ be an IM, and $\mathcal{S}, \mathcal{S}'$ be two ST databases.
For $X \in \{ID, Time, Region\}$,
if $X(\mathcal{S}) \cap X(\mathcal{S}') = \emptyset$ and $\mathcal{M}(\mathcal{S} \cup \mathcal{S}') = \mathcal{M}(\mathcal{S}) \cup \mathcal{M}(\mathcal{S}')$, then
$\mathcal{I}(\mathcal{S} \cup \mathcal{S}') = \mathcal{I}(\mathcal{S}) + \mathcal{I}(\mathcal{S}')$.

Results

	Dimensional IMs					Repair-based IMs			
	$\mathcal{I}_O$	$\mathcal{I}_T$	$\mathcal{I}_{OT}$	$\mathcal{I}_S$	$\mathcal{I}_D$	$\mathcal{I}_{id}$	$\mathcal{I}_{time}$	$\mathcal{I}_{region}$	$\mathcal{I}_{card}$
Free-Formula Independence	✓	✓	✓	✓	✓	✗	✗	✗	✓
Penalty	✗	✗	✗	✗	✓	✗	✗	✗	✗
Dominance	✓	✓	✓	✗	✓	✓	✓	✗	✓
Super-Additivity	✗	✗	✗	✗	✓	✓	✓	✓	✓
Attenuation	✗	✗	✗	✗	✗	✗	✗	✗	✗
Equal Conflict	✓	✓	✓	✗	✗	✗	✗	✗	✓
MI-Normalization	✓	✓	✓	✗	✗	✗	✗	✗	✓
MI-Separability	✗	✗	✗	✗	✓	✗	✗	✗	✗
Dimensional Penalty	✓	✓	–	✓	✓	✓*	✓*	✓	–
Dimensional Super-Additivity	✓	✓	–	✓	✓	✓	✓	✓	–
Dimensional MI-Separability	✓	✓	–	✓	✓	✗	✗	✓	–
Complexity	P	P	P	P *	P *	NP-c	NP-c	P †	P

✓*: satisfied when measures return a finite value

*: under the restriction of rectangular regions

†: for the symmetric difference metric

–: not applicable

Conclusion

- Inconsistency measures (IMs) for ST databases
  - Dimensional IMs
  - Repair-based IMs
- Postulates
  - Classical
  - Dimensional
- Postulate satisfaction
- Complexity analysis

THANKS!