Incremental Computation of Deterministic Extensions for Dynamic Argumentation Frameworks

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Argumentation in AI

- A general way for representing arguments and relationships (rebuttals) between them
- It allows representing dialogues, making decisions, and handling inconsistency and uncertainty

Abstract Argumentation Framework (AF) [Dung 1995]: arguments are abstract entities (no attention is paid to their internal structure) that may attack and/or be attacked by other arguments

Example (a simple AF)

a = Our friends will have great fun at our party on Saturday
b = Saturday will rain (according to the weather forecasting service 1)
c = Saturday will be sunny (according to the weather forecasting service 2)

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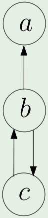
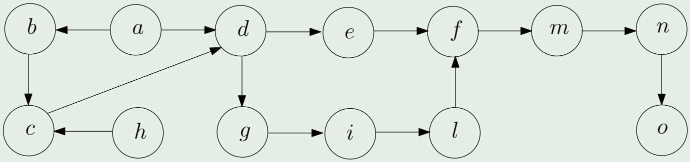


Semantics for Abstract Argumentations

- Several semantics have been proposed to identify "reasonable" sets of arguments (called *extensions*)

Example (AF $\mathcal{A}_0$)



$E_0 = \{a, h, g, e, l, m, o\}$ is an extension according to the most popular semantics, i.e. *grounded*, *complete*, *ideal*, *preferred*, *stable*, and *semi-stable*

- Extensions change if we update the initial AF by adding/removing arguments/attacks
- **Should we recompute the semantics of updated AFs from scratch?**

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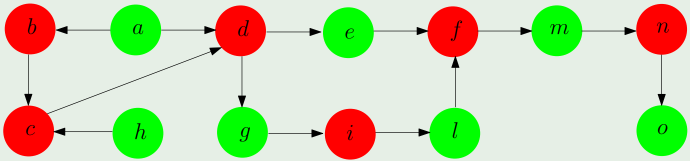

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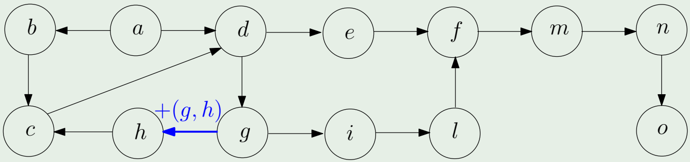

We need to recompute the extension E of the AF obtained by adding attack (g, h)

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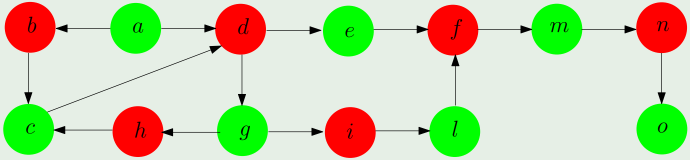

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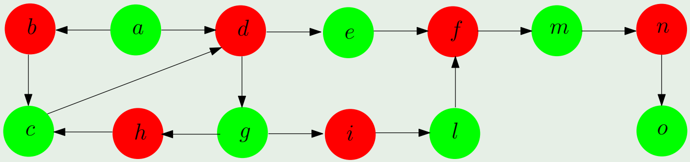

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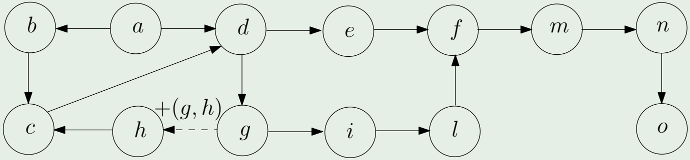
- **Should we recompute the semantics of updated AFs from scratch?**

Influenced set

- For the grounded and ideal semantics, the extension E can be efficiently computed incrementally by looking only at a small part of the AF, which is "influenced by" the update operation.

Example (AF $\mathcal{A}_0$ and updated AF $\mathcal{A} = +(g, h)(\mathcal{A}_0)$)



The influenced set is just $\{h, c\}$: only the status of h and c changes

- The influenced set refines the previously proposed set of *affected arguments* [Liao et al. 2011, Baroni et. al. 2014]
- In the example, all the arguments but a and b turn out to be affected

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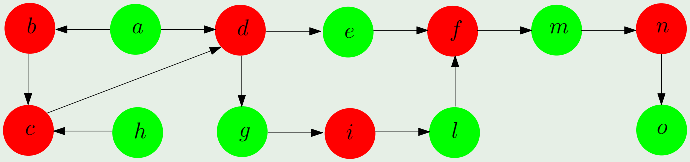

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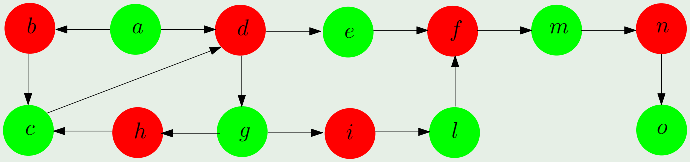

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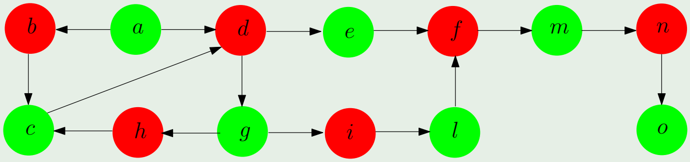

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Incremental algorithms and experiments

- We formally define the concept of *influenced set* consisting of the arguments whose status could change after an update.
- We focus on the grounded and ideal semantics, which are deterministic (admit exactly one extension)
- We present an incremental algorithm for recomputing the grounded extension; it computes the status of influenced arguments only.
- We present an incremental algorithm for the efficient recomputation of the ideal semantics which takes advantage of both the set of influenced arguments and the efficient algorithm for computing grounded extensions.
- Experimental results show the effectiveness of our approach.

Outline

1 Introduction
- Motivation
- Contributions

2 Preliminaries
- Abstract Argumentation Frameworks
- Updates

3 Algorithms
- Influenced Arguments
- Incremental Computation of Grounded Semantics
- Incremental Computation of Ideal Semantics

4 Experiments

5 Conclusions and future work
- References

Basic concepts

- An *(abstract) argumentation framework* (*AF*) is a pair $\langle A, \Sigma \rangle$, where A is a set of *arguments* and $\Sigma \subseteq A \times A$ is a set of *attacks*.

Example (AAF)

$A = \{a, b, c, d\}$
$\Sigma = \{(a, b), (b, a), (b, c), (d, c)\}$

- A set $S \subseteq A$ is *conflict-free* if there are no $a, b \in S$ such that *a attacks b*
- *S defends a* iff $\forall b \in A$ that *attacks a* there is $c \in S$ that *attacks b*
- S is *admissible* if it is conflict-free and it defends all its arguments.

Example (conflict-free and admissible sets)

- $\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, d\}$ are conflict-free
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Semantics for Abstract Argumentation

A semantics identifies "reasonable" sets of arguments, called *extensions*

- A *complete extension* ($\mathtt{co}$) is an admissible set that contains all the arguments that it defends.

A complete extension S is said to be:

- *preferred* ($\mathtt{pr}$) iff it is maximal
- *semi-stable* ($\mathtt{ss}$) iff $S \cup S^+$ is maximal (S^+ are arguments attacked by S)
- *stable* ($\mathtt{st}$) iff it attacks each argument in $A \setminus S$
- *grounded* ($\mathtt{gr}$) iff it is minimal
- *ideal* ($\mathtt{id}$) iff it is contained in every preferred extension and it is maximal

Example (semantics for AAF)

complete extensions: $\{d\}, \{a, d\}, \{b, d\}$

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Deterministic (or unique status) semantics

- All the semantics except the stable admit at least one extension
- Grounded and ideal semantics admit exactly one extension
- Semantics `gr` and `id` are called *deterministic* or *unique status*

Example (Multiple status vs unique status semantics)

3 complete extensions: $\{d\}, \{a, d\}, \{b, d\}$
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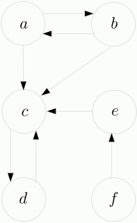

grounded extension: $\{f\}$

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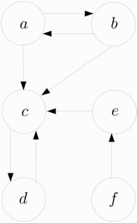

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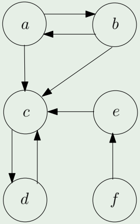

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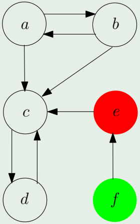

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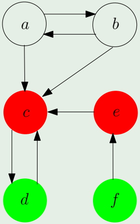
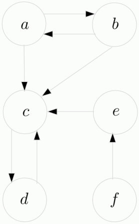


Extensions and labellings

- Semantics can be also defined in terms of *labelling*.
- Function $L : A \to \{\text{IN}, \text{OUT}, \text{UN}\}$ assigns a label to each argument
  - $L(a) = \text{IN}$ means a is accepted (i.e., *all* arguments attacking a are rejected)
  - $L(a) = \text{OUT}$ means a is rejected (i.e., an argument attacking a is accepted)
  - $L(a) = \text{UN}$ means that a is undecided
- Extension E corresponds to the labelling $L = \langle E, E^+, A \setminus (E \cup E^+) \rangle$
- Labelling L corresponds to the extension consisting of the arguments labelled as IN

Example (Two complete labellings: the grounded and the ideal labelling)



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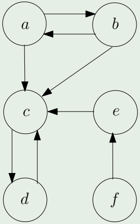

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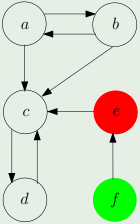

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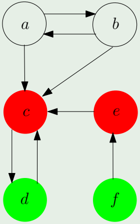
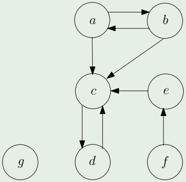


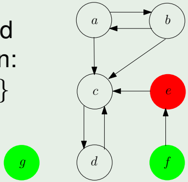
Updates

- An *update u* for an AF $\mathcal{A}_0$ consists in modifying $\mathcal{A}_0$ into an AF $\mathcal{A}$ by adding or removing arguments or attacks.
- If E_0 is an extension for $\mathcal{A}_0$ and $\mathcal{A}$ is obtained by adding (resp. removing) the set S of isolated arguments, then $E = E_0 \cup S$ (resp. $E = E_0 \setminus S$)
- We focus on the addition $+(a, b)$ and deletion $-(a, b)$ of an attack (a, b)
- $u(\mathcal{A}_0)$ denotes the application of update $u = \pm(a, b)$ to $\mathcal{A}_0$.
- Multiple updates $\{+(a_1, b_1), \ldots, +(a_n, b_n), -(a'_1, b'_1), \ldots, -(a'_m, b'_m)\}$ can be simulated by a single attack update

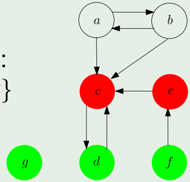
Example (Extensions/labellings after adding the isolated argument g)



grounded extension: $\{f\} \cup \{g\}$



ideal extension: $\{d, f\} \cup \{g\}$



Updates

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- If E_0 is an extension for $\mathcal{A}_0$ and $\mathcal{A}$ is obtained by adding (resp. removing) the set S of isolated arguments, then $E = E_0 \cup S$ (resp. $E = E_0 \setminus S$)
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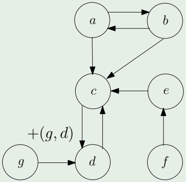
Example (Extensions/labellings after adding the attack $+(g, d)$)



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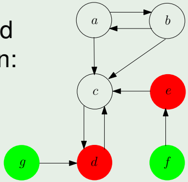

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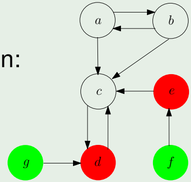

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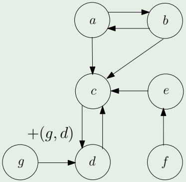

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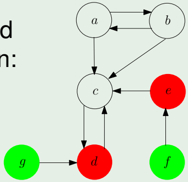

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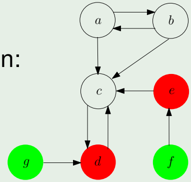

Outline

1 Introduction
- Motivation
- Contributions

2 Preliminaries
- Abstract Argumentation Frameworks
- Updates

3 Algorithms
- Influenced Arguments
- Incremental Computation of Grounded Semantics
- Incremental Computation of Ideal Semantics

4 Experiments

5 Conclusions and future work
- References

Overview of the approach

We have an initial AF $\mathcal{A}_0$, extension E_0, and updated $u = \pm(a, b)$
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1) Compute the set of arguments of $\mathcal{A}_0$ whose status can change after performing update u (*Influenced Set* $\mathcal{I}(u, \mathcal{A}_0, E_0)$)
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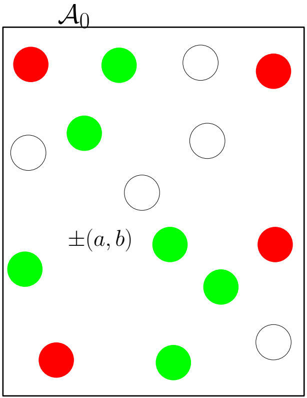

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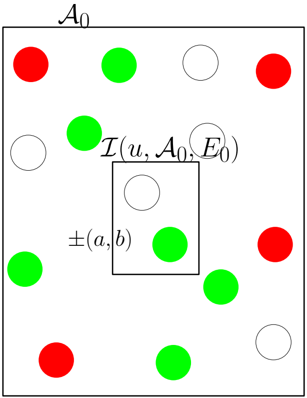

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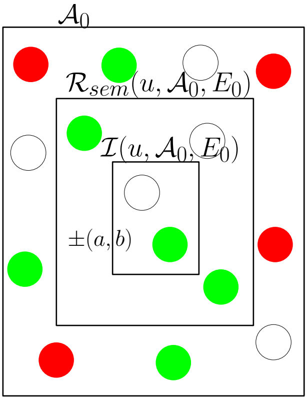

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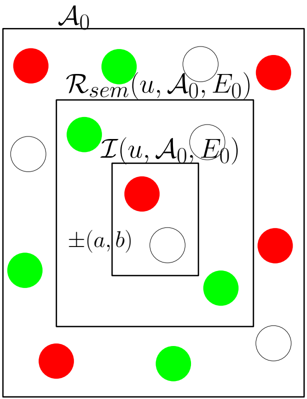

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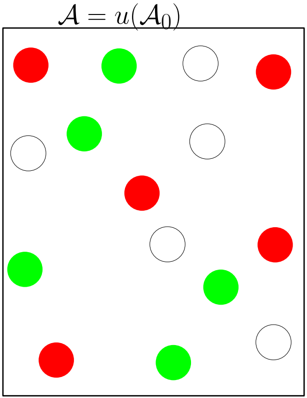

Conditions for extension/labelling preservation

- $\mathcal{E}_{\mathcal{S}}(\mathcal{A})$ denotes the set of extensions of AF $\mathcal{A}$ according to $\mathcal{S}$

Proposition (*Addition of an attack*)

Let $u = +(a, b)$ *and* $E_0 \in \mathcal{E}_{\mathcal{S}}(\mathcal{A}_0)$ *be an extension of* $\mathcal{A}_0$ *under semantics* $\mathcal{S}$, *and* L_0 *the labelling corresponding to* E_0. *Then* $E_0 \in \mathcal{E}_{\mathcal{S}}(u(\mathcal{A}_0))$ *if*

- $\mathcal{S} \in \{co, st, gr\}$ *and one of the following conditions holds:*
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  - $L_0(a) =$ IN *and* $L_0(b) =$ OUT;
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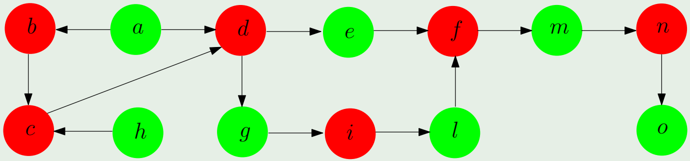

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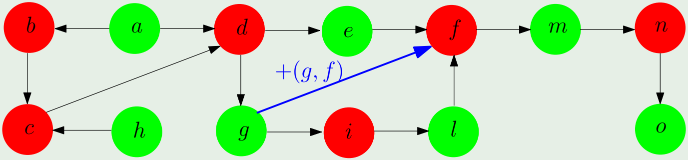

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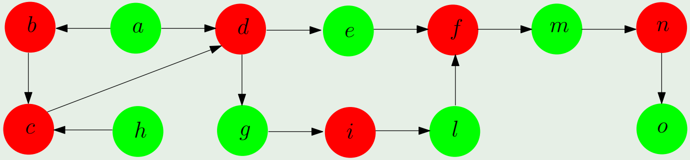

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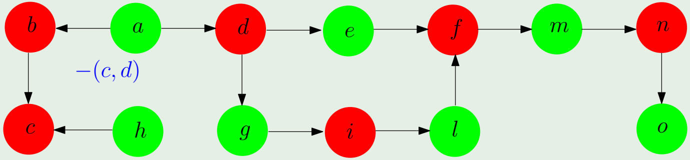

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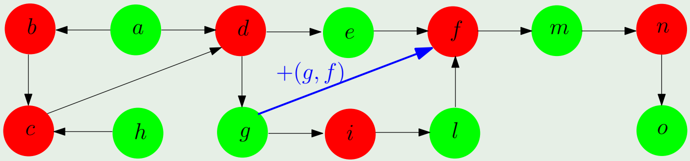
Influenced set: Intuition

- $\mathcal{I}(u, \mathcal{A}_0, E_0))$ is the *influenced set* of $u = \pm(a, b)$ w.r.t. $\mathcal{A}_0$ and E_0

1) if a condition for extension preservation holds, then $\mathcal{I}(u, \mathcal{A}_0, E_0) = \emptyset$
2) the status of an argument can change only if it is reachable from b (that is, $\mathcal{I}(u, \mathcal{A}_0, E_0) \subseteq Reach_{\mathcal{A}}(b)$)
3) if argument z is not reachable from b and $z \in E_0$, then also the status of the arguments attacked by z cannot change: their status remain OUT

Example (Set of arguments influenced by an update operation)



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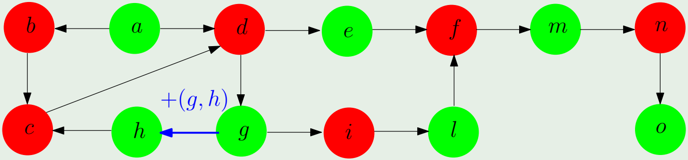

$\mathcal{I}(+(g, h), \mathcal{A}_0, E_0) \subseteq Reach_{\mathcal{A}}(h) = (A \setminus \{a, b\}) = \{c, d, e, f, g, h, i, l, m, n, o\}$
We have that $a, b \notin \mathcal{I}(+(g, h), \mathcal{A}_0, E_0)$

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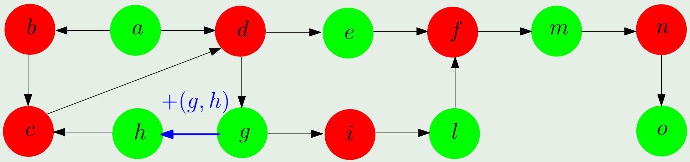

$d \notin \mathcal{I}(+(g, h), \mathcal{A}_0, E_0)$ since it is attacked by $a \in E_0$ and a is not reachable from h. Thus the arguments that can be reached only using d cannot belong to $\mathcal{I}(+(g, h), \mathcal{A}_0, E_0)$. $\rightarrow$ **The influenced set is** $\mathcal{I}(+(g, h), \mathcal{A}_0, E_0) = \{h, c\}$

Influenced set: Definition

- $\mathcal{I}(\pm(a,b),\mathcal{A}_0,E_0)$ is the set of arguments that can be reached from b without using any intermediate argument y whose status is known to be OUT because it is determined by an argument $z \in E_0$ which is not reachable from b

Definition (Influenced set)

Let $\mathcal{A} = \langle A, \Sigma \rangle$ be an AF, $u = \pm(a,b)$ an update, E an extension of $\mathcal{A}$ under a given semantics $\mathcal{S}$, and let

- $\mathcal{I}_0(u,\mathcal{A},E) = \begin{cases} \emptyset & \text{if } E \in \mathcal{E}_{\mathcal{S}}(u(\mathcal{A}))\ \ [\text{ Prop. 1/2 }] \text{ or} \\ & \exists (z,b) \in \Sigma \text{ s.t. } z \in E \wedge z \notin Reach_{\mathcal{A}}(b); \\ \{b\} & \text{otherwise}; \end{cases}$

- $\mathcal{I}_{i+1}(u,\mathcal{A},E) = \mathcal{I}_i(u,\mathcal{A},E) \cup \{y \mid \exists (x,y) \in \Sigma \text{ s.t. } x \in \mathcal{I}_i(u,\mathcal{A},E) \wedge \nexists (z,y) \in \Sigma \text{ s.t. } z \in E \wedge z \notin Reach_{\mathcal{A}}(b)\}$.

The influenced set of u w.r.t. $\mathcal{A}$ and E is $\mathcal{I}(u,\mathcal{A},E) = \mathcal{I}_n(u,\mathcal{A},E)$ such that $\mathcal{I}_n(u,\mathcal{A},E) = \mathcal{I}_{n+1}(u,\mathcal{A},E)$. □

Restricted AF for grounded semantics

- Given an AF $\mathcal{A}_0$, its grounded extension E_0, and an update $u = \pm(a, b)$, the grounded semantics is recomputed for a small part of the initial AF, called restricted AF and denoted $\mathcal{R}_{\mathrm{gr}}(u, \mathcal{A}_0, E_0)$
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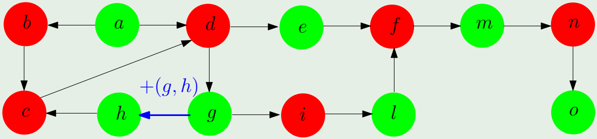

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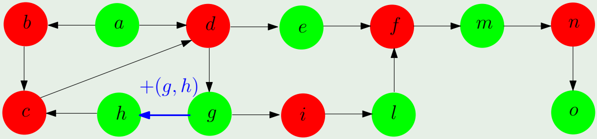

$\mathcal{I}(+(g, h), \mathcal{A}_0, E_0) = \{h, c\}$ subgraph induced by $\mathcal{I}$

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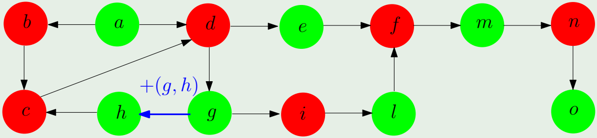

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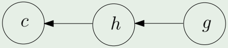

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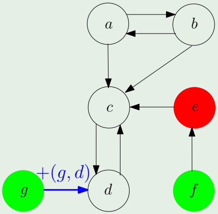
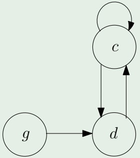
Example (Restricted AF for grounded semantics)



$\mathcal{I}(+(g, d), \mathcal{A}_0, E_0) = \{d, c\}$

Restricted AF:



Incremental algorithm for grounded semantics

Algorithm Incr-Grounded-Sem($\mathcal{A}_0, u, E_0$)

Input: AF $\mathcal{A}_0 = \langle A_0, \Sigma_0 \rangle$, $u = \pm(a, b)$, grounded extension E_0;
Output: Revised grounded extension E

```
Let S = I(u, A_0, E_0); // Compute the influenced set
Let A_d = ⟨A_d, Σ_d⟩ = R_gr(u, A_0, E_0); // Compute the restricted AF
if (A_d = ∅) then E = E_0; // If restricted AF is empty, return the initial extension E_0
else E = (E_0 \ S) ∪ IFP(A_d, E_0 ∩ A_d); // Merge E_0 with the extension of the restricted AF
```

- Let $S = \mathcal{I}(u, \mathcal{A}_0, E_0)$; // Compute the influenced set
- Let $\mathcal{A}_d = \langle A_d, \Sigma_d \rangle = \mathcal{R}_{\mathtt{gr}}(u, \mathcal{A}_0, E_0)$; // Compute the restricted AF
- **if** ($A_d = \emptyset$) **then** $E = E_0$; // If restricted AF is empty, return the initial extension E_0
- **else** $E = (E_0 \setminus S) \cup IFP(\mathcal{A}_d, E_0 \cap A_d)$; // Merge E_0 with the extension of the restricted AF

Function *IFP($\mathcal{A}, E_0$) // Incremental FixPoint*

Input: AF $\mathcal{A} = \langle A, \Sigma \rangle$, Extension E_0;
Output: Extension E

- $S_{\text{IN}} = \Delta_{\text{IN}} = \{ a \mid \nexists (c, a) \in \Sigma \}$; // Compute the starting set of arguments labelled IN
- **if** ($S_{\text{IN}} = \emptyset$) **return** S_{IN};
- $S_{\text{OUT}} = \Delta_{\text{OUT}} = \Delta^{+}_{\text{IN}}$; // Arguments attacked by Δ_{IN} are OUT
- **repeat**
  - $\Delta_{\text{IN}} = G(S_{\text{OUT}}, \Delta_{\text{OUT}}) \setminus S_{\text{IN}}$; // Infer new arguments that can be labelled IN
  - $\Delta_{\text{OUT}} = \Delta^{+}_{\text{IN}} \setminus S_{\text{OUT}}$; // New arguments labelled OUT
  - $S_{\text{IN}} = S_{\text{IN}} \cup \Delta_{\text{IN}}$; // Update the set of arguments labelled IN
  - $S_{\text{OUT}} = S_{\text{OUT}} \cup \Delta_{\text{OUT}}$; // ... and OUT
- **until** $\Delta_{\text{IN}} \subseteq E_0$ // Until no new labels (w.r.t. the initial labelling) are inferred
- **if** ($\Delta_{\text{IN}} = \emptyset$) **return** S_{IN};
- **else return** $S_{\text{IN}} \cup (E_0 \setminus (S_{\text{IN}} \cup S_{\text{OUT}}))$; // Merge the inferred labels with existing ones

Incremental algorithm for grounded semantics

Algorithm Incr-Grounded-Sem($\mathcal{A}_0, u, E_0$)

Input: AF $\mathcal{A}_0 = \langle A_0, \Sigma_0 \rangle$, $u = \pm(a, b)$, grounded extension E_0;
Output: Revised grounded extension E

```
Let S = I(u, A_0, E_0); // Compute the influenced set
Let A_d = ⟨A_d, Σ_d⟩ = R_gr(u, A_0, E_0); // Compute the restricted AF
if (A_d = ∅) then E = E_0; // If restricted AF is empty, return the initial extension E_0
else E = (E_0 \ S) ∪ IFP(A_d, E_0 ∩ A_d); // Merge E_0 with the extension of the restricted AF
```

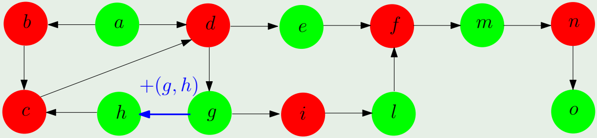
Function *IFP($\mathcal{A}, E_0$) // Incremental FixPoint*

Input: AF $\mathcal{A} = \langle A, \Sigma \rangle$, Extension E_0;
Output: Extension E

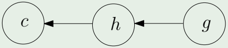
```
S_IN = Δ_IN = { a | ∄(c, a) ∈ Σ }; // Compute the starting set of arguments labelled IN
if (S_IN = ∅) return S_IN;
S_OUT = Δ_OUT = Δ_IN^+; // Arguments attacked by Δ_IN are OUT
repeat
    Δ_IN = G(S_OUT, Δ_OUT) \ S_IN; // Infer new arguments that can be labelled IN
    Δ_OUT = Δ_IN^+ \ S_OUT; // New arguments labelled OUT
    S_IN = S_IN ∪ Δ_IN; // Update the set of arguments labelled IN
    S_OUT = S_OUT ∪ Δ_OUT; // ... and OUT
until Δ_IN ⊆ E_0 // Until no new labels (w.r.t. the initial labelling) are inferred
if (Δ_IN = ∅) return S_IN;
else return S_IN ∪ (E_0 \ (S_IN ∪ S_OUT)); // Merge the inferred labels with existing ones
```

Example 1 of incremental computation

Example (From the initial extension and the update to the revised extension)



Influenced set $\mathcal{I}(+(g,h), \mathcal{A}_0, E_0) = \{h, c\}$

Restricted AF: 

Extension for the restricted AF:

Revised extension for the updated AF:

Example 1 of incremental computation

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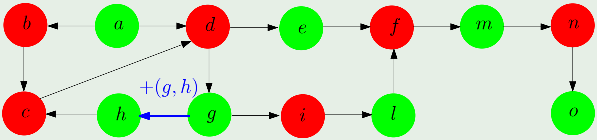

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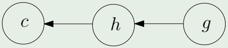
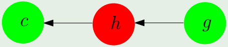
Restricted AF: 

Extension for the restricted AF: 

Revised extension for the updated AF:

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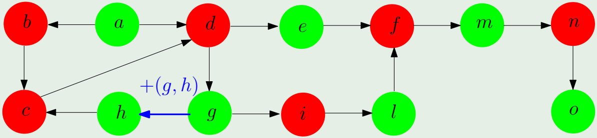

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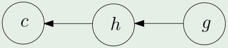
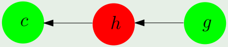
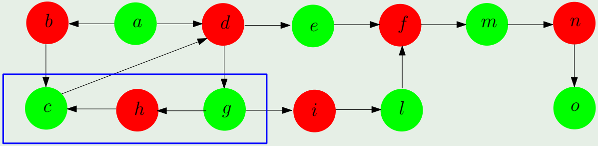
Restricted AF: 

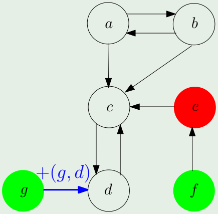
Extension for the restricted AF: 

Revised extension for the updated AF:



Example 2 of incremental computation

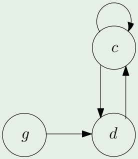
Example (From the initial extension and the update to the revised extension)



Influenced set

$\mathcal{I}(+(g,d), \mathcal{A}_0, E_0) = \{d, c\}$

Restricted AF:

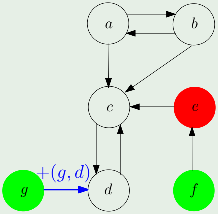


Extension for the restricted AF:

Revised extension for the updated AF:

Example 2 of incremental computation

Example (From the initial extension and the update to the revised extension)



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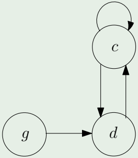
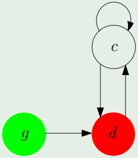
Restricted AF:



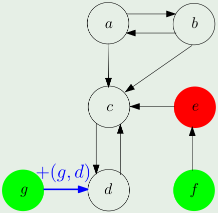
Extension for the restricted AF:



Revised extension for the updated AF:

Example 2 of incremental computation

Example (From the initial extension and the update to the revised extension)



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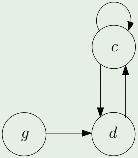

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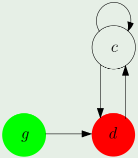
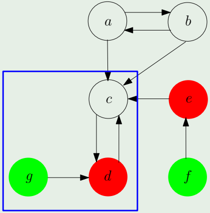


Revised extension for the updated AF:



Restricted AF for ideal semantics

- Restricted AF for ideal semantics $\mathcal{R}_{\mathtt{id}}(u, \mathcal{A}_0, E_0)$
- $\mathcal{R}_{\mathtt{gr}}(u, \mathcal{A}_0, E_0)$ consists of the subgraph of $u(\mathcal{A}_0)$ induced by $\mathcal{I}(u, \mathcal{A}_0, E_0)$
- plus additional nodes/edges representing the "external context":
  1) if there is in $u(\mathcal{A}_0)$ an edge from a node $a \notin \mathcal{I}(u, \mathcal{A}_0, E_0)$ to a node $b \in \mathcal{I}(u, \mathcal{A}_0, E_0)$, we add edge (a, b) if the status of a is IN,
  2) all nodes and edges occurring in paths (of any length) ending in $\mathcal{I}(u, \mathcal{A}_0, E_0)$ whose nodes outside $\mathcal{I}(u, \mathcal{A}_0, E_0)$ are all labeled as UN.

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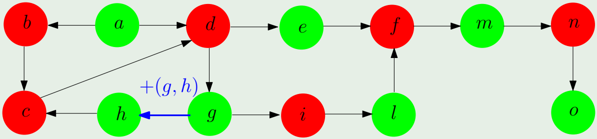

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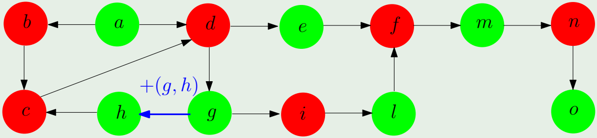

$\mathcal{I}(+(g,h), \mathcal{A}_0, E_0) = \{h, c\}$ subgraph induced by $\mathcal{I}$

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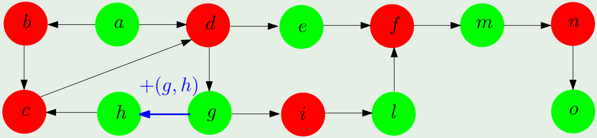

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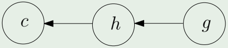

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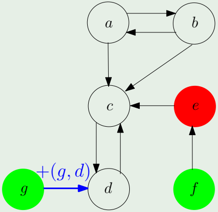

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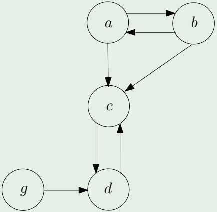

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Output: Revised ideal extension E;

Let $\mathcal{A} = u(\mathcal{A}_0)$;
$S = \mathcal{I}(u, \mathcal{A}_0, E_0)$; // Compute the influenced set
$E = E_0 \setminus S$; // The status of influenced arguments needs to be computed
if $(S = \emptyset)$ **then return** // If the influenced set is empty, done
while $(S \neq \emptyset)$ **do**
    $\mathcal{A}_d = \langle A_d, \Sigma_d \rangle = \mathcal{R}_{\mathtt{gr}}(u, \mathcal{A}_0, E)$; // Compute the restricted AF for grounded semantics
    $\Delta_{\text{IN}} = IFP(\mathcal{A}_d, E \cap A_d)$; // Computed the grounded semantics
    $S = S \setminus (\Delta_{\text{IN}} \cup \Delta_{\text{IN}}^{+})$; // Remove from decided arguments
    $E = E \cup \Delta_{\text{IN}}$; // Update the extension being computed
    $\mathcal{A}_d = \mathcal{R}_{\mathtt{id}}(u, \mathcal{A}_0, E)$; // Compute the restricted AF for ideal semantics
    Select an argument $c \in S$;
    if $\exists$ successful CWS $w \in \mathcal{CW}(c, \mathcal{A}_d, E)$ **then**
        $\Delta_{\text{IN}} = PRO(w)$; // A Coherent Winning Strategy (CWS) proves whether
        $S = S \setminus (\Delta_{\text{IN}} \cup \Delta_{\text{IN}}^{+})$; // a list of arguments belong to the ideal extension
        $E = E \cup \Delta_{\text{IN}}$;
    else $S = S \setminus \{c\}$; // Otherwise, c is not in the ideal extension

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Outline

1 Introduction
- Motivation
- Contributions

2 Preliminaries
- Abstract Argumentation Frameworks
- Updates

3 Algorithms
- Influenced Arguments
- Incremental Computation of Grounded Semantics
- Incremental Computation of Ideal Semantics

4 Experiments

5 Conclusions and future work
- References

Datasets and algorithms

Datasets

- For grounded semantics, datasets from ICCMA (International Competition on Computational Models of Argumentation)
  - `REAL` : 19 AFs $\langle A_0, \Sigma_0 \rangle$ with $|A_0| \in [5K, 100K]$ and $|\Sigma_0| \in [7K, 143K]$
  - `SYN1` : 24 AFs $\langle A_0, \Sigma_0 \rangle$ with $|A_0| \in [1K, 4K]$ and $|\Sigma_0| \in [14K, 172K]$
- For ideal semantics, `SYN2` consists of 20 AFs with $|A_0| \in \{50, 75, \ldots 175\}$

Algorithms:

- ***BaseG*** computes the grounded extension E of the updated AF $u(\mathcal{A}_0)$ from scratch: it finds the fixpoint of the characteristic function of an AF as implemented in the libraries of the *Tweety* Project
- ***BaseI*** computes the ideal extension E of the updated AF $u(\mathcal{A}_0)$ from scratch: it uses the algorithm implemented by Dung-O-Matic engine
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Datasets and algorithms

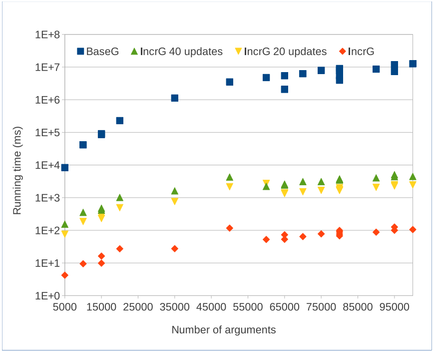
Datasets

- For grounded semantics, datasets from ICCMA (International Competition on Computational Models of Argumentation)
  - `REAL` : 19 AFs $\langle A_0, \Sigma_0 \rangle$ with $|A_0| \in [5K, 100K]$ and $|\Sigma_0| \in [7K, 143K]$
  - `SYN1` : 24 AFs $\langle A_0, \Sigma_0 \rangle$ with $|A_0| \in [1K, 4K]$ and $|\Sigma_0| \in [14K, 172K]$
- For ideal semantics, `SYN2` consists of 20 AFs with $|A_0| \in \{50, 75, \ldots 175\}$

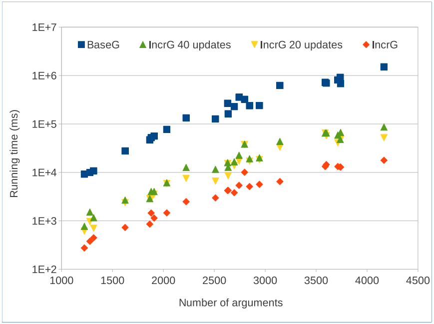
Algorithms:

- ***BaseG*** computes the grounded extension E of the updated AF $u(\mathcal{A}_0)$ from scratch: it finds the fixpoint of the characteristic function of an AF as implemented in the libraries of the *Tweety* Project
- ***BaseI*** computes the ideal extension E of the updated AF $u(\mathcal{A}_0)$ from scratch: it uses the algorithm implemented by Dung-O-Matic engine
- *Incr-Grounded-Sem* (***IncrG***) incrementally computes the grounded extension E starting from E_0 and the update
- *Incr-Ideal-Sem* (***IncrI***) incrementally computes the ideal extension E starting from E_0 and the update

Experiments for grounded semantics



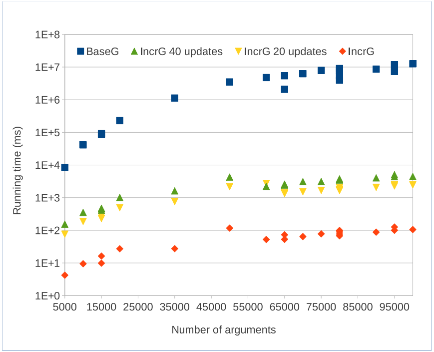
Run times (ms) of *BaseG* and *IncrG* for 1, 20, and 40 updates over REAL



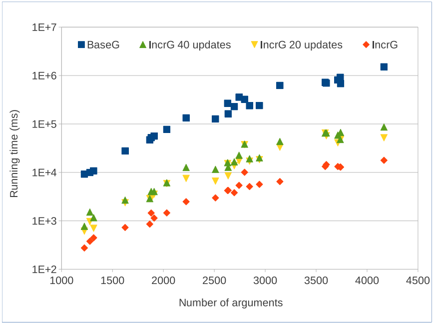
Run times (ms) of *BaseG* and *IncrG* for 1, 20, and 40 updates over SYN1

- *IncrG* and *IncrI* compute extensions of AFs updated by a set U of (simultaneous) updates by reducing the application of $U = \{+(a_1, b_1), \ldots, +(a_n, b_n), -(a'_1, b'_1), \ldots, -(a'_m, b'_m)\}$ on AF $\mathcal{A}_0$ to the application of a single attack update on an AF obtained from $\mathcal{A}_0$

Experiments for grounded semantics



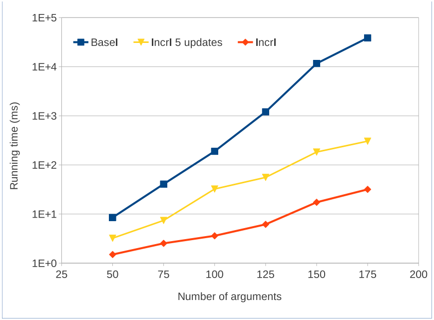
Run times (ms) of *BaseG* and *IncrG* for 1, 20, and 40 updates over `REAL`



Run times (ms) of *BaseG* and *IncrG* for 1, 20, and 40 updates over `SYN1`

- *IncrG* and *IncrI* compute extensions of AFs updated by a set U of (simultaneous) updates by reducing the application of $U = \{+(a_1, b_1), \ldots, +(a_n, b_n), -(a'_1, b'_1), \ldots, -(a'_m, b'_m)\}$ on AF $\mathcal{A}_0$ to the application of a single attack update on an AF obtained from $\mathcal{A}_0$

Experiments for ideal semantics



Run times (ms) of *BaseI* and *IncrI*
for 1 and 5 updates over SYN2

- Linear improvements for grounded semantics
- Exponential improvements for ideal semantics (whose computation from scratch is exponential)

Outline

1. Introduction
   - Motivation
   - Contributions
2. Preliminaries
   - Abstract Argumentation Frameworks
   - Updates
3. Algorithms
   - Influenced Arguments
   - Incremental Computation of Grounded Semantics
   - Incremental Computation of Ideal Semantics
4. Experiments
5. Conclusions and future work
   - References

Conclusions and future work

- We presented two incremental algorithms for computing deterministic extensions of updated AFs
- The algorithms exploit the initial extension of an AF for computing the set of arguments influenced by an update,
- and for detecting early termination conditions during the recomputation of the status of the arguments.
- The technique can be used in the case of general multiple updates.
- The experiments showed that the incremental computation outperforms that of the base (non-incremental) computation
- The definition of influenced set substantially restricts the portion of the AF to be analysed for recomputing the semantics after an update.
- Future work: application of the technique to other (multiple status) semantics.

Thank you!

... any question?

Selected References

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