Efficiently Estimating the Probability of Extensions in Abstract Argumentation

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Argumentation in AI

- A general way for representing arguments and relationships (rebuttals) between them
- It allows representing dialogues, making decisions, and handling inconsistency and uncertainty

Abstract Argumentation Framework (AAF) [Dung 1995]: arguments are abstract entities (no attention is paid to their internal structure) that may attack and/or be attacked by other arguments

Example (a simple AAF)

a = Our friends will have great fun at our party on Saturday
b = Saturday will rain (according to the weather forecasting service 1)
c = Saturday will be sunny (according to the weather forecasting service 2)

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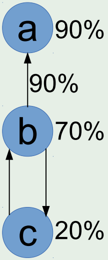
Probabilistic Abstract Argumentation Framework

- Arguments and attacks can be uncertain

Example (modelling uncertainty in our simple AAF)

there is some uncertainty

- about the fact that our friends will have fun at the party
- about the truthfulness of the weather forecasting services
- about the fact that the bad weather forecast actually entails that the party will be disliked by our friends



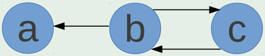
In a **Probabilistic Argumentation Framework (PrAF)** [Li et Al. 2011] both arguments and defeats are associated with probabilities

Semantics for Abstract Argumentations

- In the deterministic setting, several semantics (such as *admissible*, *stable*, *complete*, *grounded*, *preferred*, and *ideal*) have been proposed to identify "reasonable" sets of arguments

Example (AAF)

For instance, $\{a, c\}$ is admissible



- These semantics do make sense in the **probabilistic setting** too: **what is the probability that a set S of arguments is reasonable?** (according to given semantics)

Example (PrAF)

the probability that $\{a, c\}$ is admissible is 0.18

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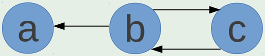

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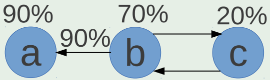

Complexity of Probabilistic Abstract Argumentation

PROB$^{sem}(S)$ is the problem of *computing the probability* $Pr^{sem}(S)$ *that a set S of arguments is reasonable according to semantics sem*

- PROB$^{sem}(S)$ is the probabilistic counterpart of the problem VER$^{sem}(S)$ of verifying whether a set S is reasonable according to semantics

sem	VER$^{sem}(S)$	PROB$^{sem}(S)$	
admissible	*PTIME*	*PTIME*	both tractable
stable	*PTIME*	*PTIME*	
complete	*PTIME*	$FP^{\#P}$-complete	from tractability to intractability
grounded	*PTIME*	$FP^{\#P}$-complete	
preferred	*coNP*-complete	$FP^{\#P}$-complete	both intractable
ideal	*coNP*-complete	$FP^{\#P}$-complete	

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Estimating the Probability of Extensions in Abstract Argumentation

In [Li et Al. 2011] a Monte-Carlo-based simulation technique for estimating the probability $\text{PROB}^{sem}(S)$, where *sem* is *complete*, *grounded*, *preferred*, is proposed.

- This method does not exploit the possibility of computing $\text{PROB}^{\text{CF}}(S)$ and $\text{PROB}^{\text{AD}}(S)$ in polynomial time.
- We propose a new method for estimating $\text{PROB}^{sem}(S)$ which:
  1. computes $\text{PROB}^{\text{CF}}(S)$ (resp. $\text{PROB}^{\text{AD}}(S)$),
  2. computes an estimate of $Pr_{\mathcal{F}}^{sem|\text{CF}}(S)$ (resp., $Pr_{\mathcal{F}}^{sem|\text{AD}}(S)$)
  3. returns $Pr_{\mathcal{F}}^{sem|\text{CF}}(S) \times Pr^{\text{CF}}(S)$ (resp., $Pr_{\mathcal{F}}^{sem|\text{AD}}(S) \times Pr^{\text{AD}}(S)$) as an estimate of $\text{PROB}^{sem}(S)$
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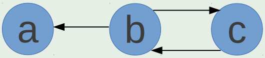
Outline

Basic concepts of Abstract Argumentation

- An *abstract argumentation framework* consists of a set A of *arguments*, and a relation $D \subseteq A \times A$, whose elements are *defeats* (or *attacks*)

Example (AAF)

$A = \{a, b, c\}$
$D = \{\langle b, a\rangle, \langle b, c\rangle, \langle c, b\rangle\}$



- A set $S \subseteq A$ of arguments is *conflict-free* if there are no $a, b \in S$ such that *a defeats b*
- An argument a is *acceptable* w.r.t. $S \subseteq A$ iff $\forall b \in A$ such that *b defeats a*, there is $c \in S$ such that *c defeats b*.

Example (conflict-free and acceptable sets)

$\{a\}, \{b\}, \{a, c\}$ are conflict-free sets;
a is acceptable w.r.t. $\{c\}$

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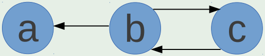

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Semantics for Abstract Argumentation

- Each semantics identifies "reasonable" sets of arguments

semantics *sem*	**A set $S \subseteq A$ of arguments is reasonable according to *sem* iff**
admissible	S is conflict-free and all its arguments are acceptable w.r.t. S
stable	S is conflict-free and S defeats each argument in $A \setminus S$
complete	S is admissible and S contains all the arguments that are acceptable w.r.t. S
grounded	S is a minimal complete set of arguments
preferred	S is a maximal admissible set of arguments

Example (semantics for AAF)

admissible sets: $\{a, c\}$, $\{b\}$, $\{c\}$, $\emptyset$
stable sets: $\{a, c\}$, $\{b\}$
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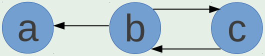

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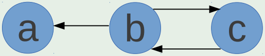

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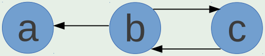

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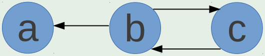

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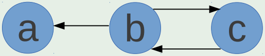

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Basics of Probabilistic Argumentation

- A *PrAF* is a tuple $\langle A, P_A, D, P_D \rangle$ where
  - $\langle A, D \rangle$ is an *AAF*, and
  - P_A and P_D are functions assigning a probability value to each argument in A and defeat in D
- $P_A(a)$ represents the probability that argument a actually occurs
- $P_D(\langle a, b \rangle)$ represents the conditional probability that a defeats b given that both a and b occur

Example (probabilities of arguments and defeats)

$P_A(a) = .9$ $\quad$ $P_D(\langle b, a \rangle) = .9$
$P_A(b) = .7$ $\quad$ $P_D(\langle b, c \rangle) = 1$
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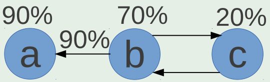

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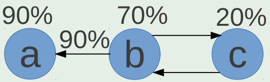

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Meaning of a probabilistic argumentation framework

- The meaning of a PrAF is given in terms of possible worlds
- A possible world represents a (deterministic) scenario consisting of some subset of the arguments and defeats of the PrAF
- given a PrAF $\mathcal{F} = \langle A, P_A, D, P_D \rangle$, a possible world w for $\mathcal{F}$ is an AAF $\langle A', D' \rangle$ such that $A' \subseteq A$ and $D' \subseteq D \cap (A' \times A')$.

Example (some possible worlds)

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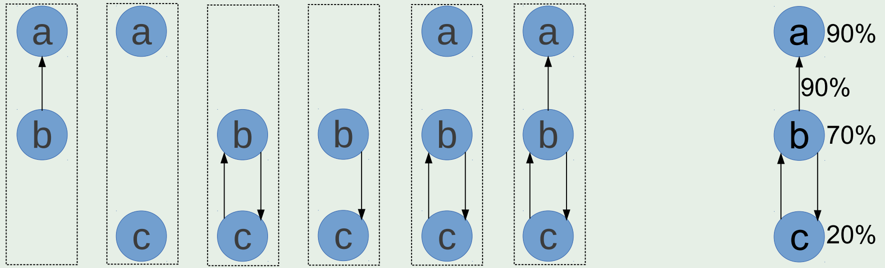

Probability of reasonable sets

- An interpretation $\mathcal{I}$ for a PrAF is a probability distribution over the set of possible worlds
- possible world w is assigned by $\mathcal{I}$ the probability $\mathcal{I}(w)$ equal to:

$$\prod_{a \in Arg(w)} P_A(a) \times \prod_{a \in A \setminus Arg(w)} (1 - P_A(a)) \times \prod_{\delta \in Def(w)} P_D(\delta) \times \prod_{\delta \in \overline{D}(w) \setminus Def(w)} (1 - P_D(\delta))$$

where $\overline{D}(w) = D \cap (Arg(w) \times Arg(w))$ is the set of defeats that may appear in w

- The probability $Pr^{sem}(S)$ that a set S of arguments is reasonable according to a given semantics *sem* is defined as *the sum of the probabilities of the possible worlds w for which S is reasonable according to sem*

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Outline

Estimating Pr^{sem}: The state of the art approach

Algorithm (A1)

State-of-the-art algorithm for approximating $Pr_{\mathcal{F}}^{sem}(S)$
Input: $\mathcal{F} = \langle A, P_A, D, P_D \rangle$; *S* ⊆ *A; sem; An error level* ϵ; *A confidence level* $z_{1-\alpha/2}$
Output: $\widehat{Pr}_{\mathcal{F}}^{sem}(S)$ *s.t.* $Pr_{\mathcal{F}}^{sem}(S) \in [\widehat{Pr}_{\mathcal{F}}^{sem}(S) - \epsilon,\ \widehat{Pr}_{\mathcal{F}}^{sem}(S) + \epsilon]$ *with confidence* $z_{1-\alpha/2}$

$success = samples = maxsamples = 0$;
do
 $Arg = Def = \emptyset$

 if *S is an extension for* $\langle Arg, Def \rangle$ *according to sem* ***then*** *success=success+1;*
 samples=samples+1; update maxsamples
while $samples \leq maxsamples$
return $\frac{success}{samples}$

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    success = samples = maxsamples = 0;
    do
        Arg = Def = ∅
        for each a ∈ A do
            With probability P_A(a) do Arg = Arg ∪ {a}

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    for each ⟨a, b⟩ ∈ D s.t. a, b ∈ Arg do
        With probability P_D(⟨a, b⟩) do Def = Def ∪ {⟨a, b⟩}
    if S is an extension for ⟨Arg, Def⟩ according to sem then success=success+1;
    samples=samples+1; update maxsamples
while samples ≤ maxsamples
return success/samples
```

Estimating $Pr_{\mathcal{F}}^{sem}(S)$ by sampling AAFs wherein S is conflict-free

Algorithm (A2)

Compute $Pr_{\mathcal{F}}^{cf}(S)$
$success = samples = maxsamples = 0$;
do
  $Arg = S$; $Def = \emptyset$;

  if *S is an extension for* $\langle Arg, Def\rangle$ *according to sem* ***then*** *success=success+1;*
  samples=samples+1; update maxsamples
while $samples \leq maxsamples$
return $\frac{success}{samples} \cdot Pr_{\mathcal{F}}^{cf}(S)$

Estimating $Pr^{sem}_{\mathcal{F}}(S)$ by sampling AAFs wherein S is conflict-free

Algorithm (A2)

Compute $Pr^{cf}_{\mathcal{F}}(S)$
$success = samples = maxsamples = 0$;
do
  $Arg = S$; $Def = \emptyset$;
  for each $a \in A \setminus S$ ***do***
    With probability $Pr\,(a|\text{CF})$ ***do*** $Arg = Arg \cup \{a\}$

  if *S is an extension for* $\langle Arg, Def\rangle$ *according to sem* ***then*** *success=success+1;*
  samples=samples+1; update maxsamples
while $samples \leq maxsamples$
return $\frac{success}{samples} \cdot Pr^{cf}_{\mathcal{F}}(S)$

Estimating $Pr^{sem}_{\mathcal{F}}(S)$ by sampling AAFs wherein S is conflict-free

Algorithm (A2)

Compute $Pr^{cf}_{\mathcal{F}}(S)$
$success = samples = maxsamples = 0;$
do
  $Arg = S;$ $Def = \emptyset;$
  for each $a \in A \setminus S$ ***do***
    With probability $Pr\,(a|\text{CF})$ ***do*** $Arg = Arg \cup \{a\}$
  for each $\langle a, b\rangle \in D$ *such that* $a, b \in Arg$ ***do***
    if $a \notin S \vee b \notin S$ ***then***
      With probability $Pr\,(\langle a, b\rangle\,|\text{CF})$ ***do*** $Def = Def \cup \{\langle a, b\rangle\}$
  if *S is an extension for* $\langle Arg, Def\rangle$ *according to sem* ***then*** *success=success+1;*
  samples=samples+1; update maxsamples
while $samples \leq maxsamples$
return $\frac{success}{samples} \cdot Pr^{cf}_{\mathcal{F}}(S)$

Estimating $Pr_{\mathcal{F}}^{sem}(S)$ by sampling AAFs wherein S is admissible

Algorithm (A3)

$success = samples = maxsamples = 0;$

Compute $Pr_{\mathcal{F}}^{ad}(S)$

do

$Arg = S; Def = \emptyset; defeatS = \emptyset;$

if *S is an extension for* $\langle Arg, Def \rangle$ *according to sem* ***then*** *success=success+1;*

samples=samples+1; update maxsamples

while $samples \leq maxsamples$

return $\frac{success}{samples} \cdot Pr_{\mathcal{F}}^{ad}(S)$

Estimating $Pr^{sem}_{\mathcal{F}}(S)$ by sampling AAFs wherein S is admissible

Algorithm (A3)

success $=$ *samples* $=$ *maxsamples* $= 0$;
Compute $Pr^{ad}_{\mathcal{F}}(S)$
do
  $Arg = S$; $Def = \emptyset$; $defeatS = \emptyset$;
  for each $a \in A \setminus S$ ***do***
    With probability $Pr(a|\text{AD})$ ***do***
      $Arg = Arg \cup \{a\}$
      With probability $Pr\,(a \rightarrow S|\text{AD} \wedge a)$ ***do***
        $Def = Def \cup$ *generateAtLeastOneDefeatAndDefend*$(\mathcal{F}, \langle Arg, Def\rangle, S, a)$
        $defeatS = defeatS \cup \{a\}$

  if S *is an extension for* $\langle Arg, Def\rangle$ *according to sem* ***then*** *success=success+1;*
  samples=samples+1; update maxsamples
while *samples* $\leq$ *maxsamples*
return $\frac{success}{samples} \cdot Pr^{ad}_{\mathcal{F}}(S)$

Estimating $Pr^{sem}_{\mathcal{F}}(S)$ by sampling AAFs wherein S is admissible

Algorithm (A3)

success $=$ *samples* $=$ *maxsamples* $= 0$;
Compute $Pr^{ad}_{\mathcal{F}}(S)$
do
  $Arg = S$; $Def = \emptyset$; $defeatS = \emptyset$;
  for each $a \in A \setminus S$ ***do***
    With probability $Pr(a|\text{AD})$ ***do***
      $Arg = Arg \cup \{a\}$
      With probability $Pr\,(a \rightarrow S|\text{AD} \wedge a)$ ***do***
        $Def = Def \cup$ *generateAtLeastOneDefeatAndDefend*$(\mathcal{F}, \langle Arg, Def\rangle, S, a)$
        $defeatS = defeatS \cup \{a\}$
  for each $\langle a, b\rangle \in D$ *s.t.* $(a, b \in Arg \setminus S) \vee (a \in S \wedge b \in Arg \setminus S \wedge b \notin defeatS)$ ***do***
    With probabilty $Pr(\langle a, b\rangle|\text{AD} \wedge b \not\rightarrow S)$ ***do*** $Def = Def \cup \{\langle a, b\rangle\}$
  if S *is an extension for* $\langle Arg, Def\rangle$ *according to sem* ***then*** *success=success+1;*
  samples=samples+1; update maxsamples
while *samples* $\leq$ *maxsamples*
return $\frac{success}{samples} \cdot Pr^{ad}_{\mathcal{F}}(S)$

Outline

Theoretical analysis of the efficiency of A2 and A3

Theorem

Let $z_{1-\alpha/2}$ be a confidence level, ϵ be an error level and let n_1, n_2 and n_3 be the number of Monte-Carlo iterations of A1, A2, and A3, respectively.

Let i_1, i_2, i_3, i_4 and i_5 be the following inequalities:

$$(i_1)\ Pr^{sem}(S) \geq k \cdot \epsilon, \quad (i_2)\ Pr^{sem|\mathrm{CF}}(S) \geq k' \cdot \overline{\epsilon}, \quad (i_3)\ Pr^{cf}(S) \leq 1 - \frac{2}{k'},$$
$$(i_4)\ Pr^{sem|\mathrm{AD}}(S) \geq k'' \cdot \overline{\epsilon}, \quad (i_5)\ Pr^{ad}(S) \leq 1 - \frac{2}{k''}.$$

- *If there exist k and k' greater than 1 such that i_1, i_2 and i_3 hold, then* $n_2 \leq n_1 \cdot \frac{k\cdot(k'+1)}{(k-1)\cdot k'} \cdot \mathrm{Pr}^{cf}_{\mathcal{F}}(S)$, *with confidence level* $z^2_{1-\alpha/2}$.
- *If there exist k and k'' greater than 1 such that i_1, i_4 and i_5 hold, then* $n_3 \leq n_1 \cdot \frac{k\cdot(k''+1)}{(k-1)\cdot k''} \cdot \mathrm{Pr}^{ad}_{\mathcal{F}}(S)$, *with confidence level* $z^2_{1-\alpha/2}$.

Experimental validation: data sets

- We performed experiments on 75 PrAFs each obtained considering a set A of arguments whose size ranges from 12 to 40.
- For each $|A|$, we considered 5 PrAFs having different sets of defeats.
- For each of the so obtained PrAFs, we considered 5 sets S of arguments, whose size was chosen in the interval $[20\%, 40\%] \cdot |A|$, and such that $Pr_{\mathcal{F}}^{\mathtt{cf}}(S)$ and $Pr_{\mathcal{F}}^{\mathtt{ad}}(S)$ ranged in the interval $[.5, .8]$ and $[.4, .7]$, respectively.

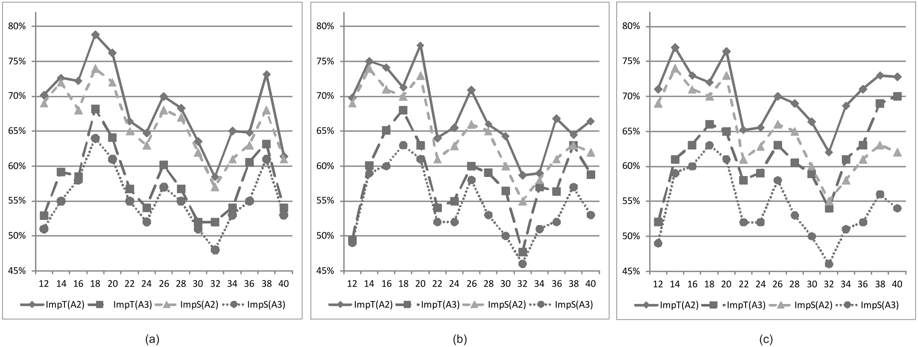
Experimental validation: performace measures

- $ImpS(A2) = \frac{samples(A2)}{samples(A1)}$ and $ImpS(A3) = \frac{samples(A3)}{samples(A1)}$, for measuring the improvement of *A2* and *A3* w.r.t. *A1*, in terms of number of generated samples;
- $ImpT(A2) = \frac{time(A2)}{time(A1)}$ and $ImpT(A3) = \frac{time(A3)}{time(A1)}$, for measuring the improvement of *A2* and *A3* w.r.t. *A1*, in terms of execution time.

where $samples(Ak)$ and $time(Ak)$, with $k \in \{1, 2, 3\}$, are the average number of samples and the average execution time of the runs of algorithm Ak, respectively.

Experimental validation: results

- On average, *ImpT*(*A*2) is equal to 70%, and *ImpS*(*A*2) is equal to 65%.
- On average, *ImpT*(*A*3) is equal to 60%, and *ImpS*(*A*3) is equal to 55%.



Improvements of A2 and A3 vs A1 for (a) complete, (b) grounded, (c) preferred semantics.

Outline

Conclusions and future work

- In this paper, we focused on estimating the probability $Pr_{\mathcal{F}}^{sem}(S)$ that a set S of arguments is an extension for a $\mathcal{F}$ according to a semantics *sem*, where *sem* is the *complete*, the *grounded*, or the *preferred* semantics.
- In particular, we proposed two algorithms for estimating $Pr_{\mathcal{F}}^{sem}(S)$, which outperform the state-of-the-art algorithm proposed in [Li et Al. 2011], both in terms of number of generated samples and evaluation time.
- Future work will be devoted to:
  - experimentally characterizing, on larger data sets, when A2 is preferable to A3,
  - applying the proposed algorithms to other semantics (e.g. the ideal set semantics) for which computing Pr^{sem} is hard.

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Thank you!

... any question?

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How to assign probabilities

- probability theory is recognized as a fundamental tool to model uncertainty
- The issue of how to assign probabilities to arguments and defeats in abstract argumentation, with particular reference to the PrAF proposed in [Li et Al. 2011], has been investigated in [Hunter 2012, Hunter 2013], where a connection among argumentation theory, classical logic, and probability theory was investigated
- In this paper, we do not address this issue, but, assuming that the probabilities of arguments and defeats are given, we tackle the probabilistic counterpart of the problem $\textsc{Ver}^{sem}(S)$

Other approaches to model uncertainty

- Besides the approaches that model uncertainty in AAFs by relying on probability theory, many proposals have been made where uncertainty is represented by exploiting weights or preferences on arguments and/or defeats, or by relying on the possibility theory
- Although the approaches based on weights, preferences, possibilities, or probabilities to model uncertainty have been proved to be effective in different contexts, there is no common agreement on what kind of approach should be used in general
- we believe that the probability-based approaches may take advantage from relying on a well-established and well-founded theory
- our complexity characterization, along with that of other approaches, may help in deciding what approach is better from a computational point of view

Estimating $Pr^{sem}_{\mathcal{F}}(S)$ by sampling AAFs wherein S is conflict-free

Lemma

Given a PrAF $\mathcal{F} = \langle A, P_A, D, P_D \rangle$ and a set $S \subseteq A$ of arguments, then

- $\forall a \in S$, $Pr(a|\text{CF})$=1; $\forall a \in A \setminus S$, $Pr(a|\text{CF})$=$P_A(a)$;
- $\forall \langle a, b \rangle \in D$ *such that* $a, b \in S$, $Pr(\langle a, b \rangle | \text{CF}) = 0$;
- $\forall \langle a, b \rangle \in D \setminus \{\langle a, b \rangle \in D \text{ s.t. } a, b \in S\}$, $Pr(\langle a, b \rangle | \text{CF}) = P_D(\langle a, b \rangle)$.

Theorem

Let ϵ be an error level, and $z_{1-\alpha/2}$ a confidence level. The estimate $\widehat{Pr}^{sem}_{\mathcal{F}}(S)$ returned by Algorithm 2 is such that $Pr^{sem}_{\mathcal{F}}(S) \in [\widehat{Pr}^{sem}_{\mathcal{F}}(S) - \epsilon, \ \widehat{Pr}^{sem}_{\mathcal{F}}(S) + \epsilon]$ with confidence level $z_{1-\alpha/2}$.

Estimating $Pr^{sem}_{\mathcal{F}}(S)$ by sampling AAFs wherein S is admissible

Fact ($\mathbf{Pr}^{ad}_{\mathcal{F}}(S)$)

$Pr^{ad}_{\mathcal{F}}(S) = Pr^{cf}_{\mathcal{F}}(S) \cdot \prod_{d \in A \setminus S} \big(P_1(S,d) + P_2(S,d) + P_3(S,d)\big)$, *where:*

- $P_1(S,d) = 1 - P_A(d)$, *i.e., the probability that d is false.*
- $P_2(S,d) = P_A(d) \cdot \prod_{\substack{\langle d,b\rangle \in D \\ \wedge b \in S}} \big(1 - P_D(\langle d,b\rangle)\big)$,
  i.e., the probability that d is true but do not attack any argument in S.
- $P_3(S,d) = P_A(d) \cdot \Big(1 - \prod_{\substack{\langle d,b\rangle \in D \\ \wedge b \in S}} \big(1 - P_D(\langle d,b\rangle)\big)\Big) \cdot \Big(1 - \prod_{\substack{\langle a,d\rangle \in D \\ \wedge a \in S}} \big(1 - P_D(\langle a,d\rangle)\big)\Big)$,
  i.e., the probability that d is true, d attack an argument in S but it is counterattacked by an argument in S.

Estimating $Pr^{sem}_{\mathcal{F}}(S)$ by sampling AAFs wherein S is admissible

Lemma

Given a PrAF $\mathcal{F} = \langle A, P_A, D, P_D \rangle$ and a set $S \subseteq A$ of arguments, then

- $\forall a \in S,\ Pr(a|\text{AD}){=}1;$
- $\forall a \in A \setminus S,\ Pr(a|\text{AD}){=}\frac{P_2(S,a)+P_3(S,a)}{P_1(S,a)+P_2(S,a)+P_3(S,a)};$
- $\forall \langle a, b \rangle \in D$ s.t. $a, b \in S,\ Pr(\langle a, b \rangle|\text{AD}) = 0;$
- $\forall \langle a, b \rangle \in D$ s.t. $a, b \in A \setminus S,\ Pr(\langle a, b \rangle|\text{AD} \wedge b \not\rightarrow S) = P_D(\langle a, b \rangle);$
- $\forall a \in A \setminus S,\ \ Pr(a \rightarrow S|\text{AD} \wedge a) = \frac{P_3(S,a)}{P_2(S,a)+P_3(S,a)};$
- $\forall \langle a, b \rangle \in D$ s.t. $a \in S \wedge b \in A \setminus S,\ Pr(\langle a, b \rangle|\text{AD} \wedge b \not\rightarrow S) = P_D(\langle a, b \rangle).$

where $P_1(S, a)$, $P_2(S, a)$, and $P_3(S, a)$ are defined as in Fact ($\mathbf{Pr}^{ad}_{\mathcal{F}}(S)$).

Estimating $Pr_{\mathcal{F}}^{sem}(S)$ by sampling AAFs wherein S is admissible

Theorem

Let ϵ be an error level, and $z_{1-\alpha/2}$ a confidence level. The estimate $\widehat{Pr}_{\mathcal{F}}^{sem}(S)$ returned by Algorithm 3 is such that $Pr_{\mathcal{F}}^{sem}(S) \in [\widehat{Pr}_{\mathcal{F}}^{sem}(S)-\epsilon,\ \widehat{Pr}_{\mathcal{F}}^{sem}(S)+\epsilon]$ with confidence level $z_{1-\alpha/2}$.